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Bartosz Łanucha and Małgorzata Michalska

WHEN IS AN ASYMMETRIC TRUNCATED HANKEL OPERATOR EQUAL TO THE ZERO OPERATOR?

Summary

In this paper we introduce the class of asymmetric truncated Hankel operators. We then describe symbols of those asymmetric truncated Hankel operators which are equal to the zero operator.

Keywords and phrases: model space, truncated Toeplitz operator, truncated Hankel operator, asymmetric truncated Hankel operator

1. Introduction

Let H^2 denote the space of functions analytic in the unit disk $\mathbb{D} = \{z : |z| < 1\}$ and such that their Maclaurin coefficients are square summable. The Hardy space H^2 can be identified via boundary values with the closed linear span of the analytic polynomials in $L^2 := L^2(\partial\mathbb{D})$. Additionally, let P denote the orthogonal projection from L^2 onto H^2.

Let $\alpha \in H^\infty = H^2 \cap L^\infty$ be such that $|\alpha| = 1$ a.e. on $\partial\mathbb{D}$. Then α is called an inner function. The corresponding model space K_α is defined by

$$K_\alpha = H^2 \ominus \alpha H^2.$$

Since K_α is a closed subspace of H^2, the point evaluation functional $f \mapsto f(w)$ is bounded on K_α for every $w \in \mathbb{D}$. Moreover,

$$f(w) = \langle f, k_w^\alpha \rangle,$$

where the reproducing kernel k_w^α is given by

$$k_w^\alpha(z) = \frac{1 - \overline{\alpha(w)}\alpha(z)}{1 - \overline{w}z}, \quad w, z \in \mathbb{D}.$$

Since each k_w^α is bounded in z, the set $K_\alpha^\infty = K_\alpha \cap H^\infty$ is dense in K_α.

For the last ten years the class of compressions of classical Toeplitz operators to model spaces has been extensively studied (see [9] for more references). Recall that for $\varphi \in L^\infty$ the classical Toeplitz operator T_φ is defined on H^2 by

$$T_\varphi f = P(\varphi f).$$

If $\varphi \in L^2$, then the above definition gives a densely defined operator. It is known that T_φ is bounded if and only if $\varphi \in L^\infty$.

A truncated Toeplitz operator (TTO) A_φ^α with symbol $\varphi \in L^2$ is the compression of T_φ to the model space K_α. More precisely,

$$A_\varphi^\alpha f = P_\alpha(\varphi f), \quad f \in K_\alpha^\infty,$$

where P_α is the orthogonal projection from L^2 onto K_α. The operator A_φ^α is densely defined but, unlike T_φ, it can be bounded for an unbounded symbol φ.

The study of truncated Toeplitz operators began in 2007 with D. Sarason's paper [11]. Recently, the authors in [4] and [5, 6] introduced a more general class of operators, the so-called asymmetric truncated Toeplitz operators.

Let α, β be two inner functions and let $\varphi \in L^2$. An asymmetric truncated Toeplitz operator (ATTO) $A_\varphi^{\alpha,\beta}$ is the operator from K_α into K_β defined by

$$A_\varphi^{\alpha,\beta} f = P_\beta(\varphi f), \quad f \in K_\alpha^\infty.$$

Closely related to Toeplitz operators are Hankel operators on H^2. A Hankel operator H_φ, $\varphi \in L^\infty$, can be defined on H^2 by

$$H_\varphi f = J(I - P)(\varphi f),$$

where J is the „flip" operator given by

$$Jf(z) = \overline{z} f(\overline{z}), \quad |z| = 1.$$

For $\varphi \in L^2$ this definition produces a densely defined operator. Truncated versions of Hankel operators were introduced by C. Gu in [2]. Here we begin the study of asymmetric truncated Hankel operators.

Let α, β be two inner functions. An asymmetric truncated Hankel operator (ATHO) $B_\varphi^{\alpha,\beta}$ with symbol $\varphi \in L^2$ is the operator from K_α into K_β defined by

$$B_\varphi^{\alpha,\beta} f = P_\beta J(I - P)(\varphi f), \quad f \in K_\alpha^\infty.$$

Let

$$\mathcal{H}(\alpha, \beta) = \{B_\varphi^{\alpha,\beta} \colon \ \varphi \in L^2 \text{ and } B_\varphi^{\alpha,\beta} \text{ is bounded}\},$$

and respectively $\mathcal{H}(\alpha) = \mathcal{H}(\alpha, \alpha)$.

It is known that the classical Toeplitz operator is uniquely determined by its symbol. In other words, $T_\varphi = 0$ if and only if $\varphi = 0$. This is not the case for TTO's and ATTO's. Namely, $A_\varphi^\alpha = 0$ if and only if $\varphi \in \overline{\alpha H^2} + \alpha H^2$ [11] and $A_\varphi^{\alpha,\beta} = 0$ if and only if $\varphi \in \overline{\alpha H^2} + \beta H^2$ [10]. As for Hankel operators, $H_\varphi = 0$ if and only if

$\varphi \in H^2$. By the result of C. Gu [2], $B_\varphi^\alpha = 0$ if and only if $\varphi \in H^2 + \overline{\alpha\alpha^\# H^2}$, where $\alpha^\#(z) = \overline{\alpha(\overline{z})}$. In this paper we show that $B_\varphi^{\alpha,\beta} = 0$ if and only if $\varphi \in H^2 + \overline{\alpha\beta^\# H^2}$, where $\beta^\#(z) = \overline{\beta(\overline{z})}$.

Note that if β is an inner function, then so is $\beta^\#$. Moreover, the operator $J^\# : L^2 \to L^2$,

$$J^\# f(z) = f^\#(z) = \overline{f(\overline{z})}, \quad |z| = 1,$$

is an antilinear isometric involution on L^2 (an operator with these properties is called a conjugation) and it preserves H^2. Furthermore, the conjugation $J^\#$ transforms K_α onto $K_{\alpha^\#}$ [7, Lem. 4.4]. Another conjugation on L^2, one that is associated with an inner function α, can be defined by

$$C_\alpha f(z) = \alpha(z)\overline{zf(z)}, \quad |z| = 1.$$

It is easy to verify that C_α is an involutive isometry which preserves K_α (see [11, Subsection 2.3]).

Before we proceed, a short remark about the definition of ATHO's is in order. Some authors (see for example [1, 3]) define a THO as the operator $\Gamma_\varphi^\alpha : K_\alpha \to \overline{zK_\alpha}$ as follows

$$\Gamma_\varphi^\alpha f = P_{\overline{\alpha}}(\varphi f), \qquad f \in K_\alpha^\infty,$$

where $P_{\overline{\alpha}}$ is the orthogonal projection from L^2 onto $\overline{zK_\alpha} = \{\overline{zf} \colon f \in K_\alpha\}$. So an ATHO could also be defined as the operator from K_α into $\overline{zK_\beta}$ given by

$$\Gamma_\varphi^{\alpha,\beta} f = P_{\overline{\beta}}(\varphi f), \qquad f \in K_\alpha^\infty.$$

However, if $\Gamma_\varphi^{\alpha,\beta}$ is as above, then $J\Gamma_\varphi^{\alpha,\beta}$ acts from K_α into $K_{\beta^\#}$ and for each $f \in K_\alpha^\infty$, $g \in K_{\beta^\#}^\infty$ (note that if $g \in K_{\beta^\#}$, then $Jg \in \overline{zK_\beta}$),

$$\left\langle J\Gamma_\varphi^{\alpha,\beta} f, g \right\rangle = \left\langle P_{\overline{\beta}}(\varphi f), Jg \right\rangle = \left\langle (I - P)(\varphi f), Jg \right\rangle = \left\langle B_\varphi^{\alpha,\beta^\#} f, g \right\rangle.$$

Thus $\Gamma_\varphi^{\alpha,\beta} = JB_\varphi^{\alpha,\beta^\#}$ and, in particular,

$$\Gamma_\varphi^\alpha = JB_\varphi^{\alpha,\alpha^\#}.$$

These two definitions are therefore equivalent.

2. The symbols of zero ATHO's

Theorem 2.1. *Let α, β be two nonconstant inner functions and let $B_\varphi^{\alpha,\beta} : K_\alpha \to K_\beta$ be a bounded asymmetric truncated Hankel operator with $\varphi \in L^2$. Then $B_\varphi^{\alpha,\beta} = 0$ if and only if $\varphi \in H^2 + \overline{\alpha\beta^\# H^2}$, where $\beta^\#(z) = \overline{\beta(\overline{z})}$.*

We first prove the following.

Proposition 2.2. *Let α, β be two nonconstant inner functions with $\alpha(0) = \beta(0) = 0$ and let $B_{\varphi}^{\alpha,\beta} : K_\alpha \to K_\beta$ be a bounded asymmetric truncated Hankel operator with $\varphi \in L^2$. Then $B_{\varphi}^{\alpha,\beta} = 0$ if and only if $\varphi \in H^2 + \overline{\alpha\beta^{\#} H^2}$, where $\beta^{\#}(z) = \overline{\beta(\overline{z})}$.*

Proof. We first prove that if $\varphi \in H^2 + \overline{\alpha\beta^{\#} H^2}$, then $B_{\varphi}^{\alpha,\beta} = 0$. Clearly, $B_{\varphi}^{\alpha,\beta} = 0$ whenever $\varphi \in H^2$. Moreover, $B_{\varphi}^{\alpha,\beta} = 0$ also for $\varphi \in \overline{\alpha\beta^{\#} H^2}$. Indeed, if $\varphi = \overline{\alpha\beta^{\#}\psi}$, $\psi \in H^2$, then for $f \in K_\alpha^\infty$, $g \in K_\beta^\infty$,

$$\begin{aligned}\left\langle B_{\varphi}^{\alpha,\beta} f, g\right\rangle &= \left\langle B_{\overline{\alpha\beta^{\#}\psi}}^{\alpha,\beta} f, g\right\rangle = \left\langle \overline{\alpha\beta^{\#}\psi} f, Jg\right\rangle = \left\langle J\left(\overline{\alpha\beta^{\#}\psi} f\right), g\right\rangle \\ &= \left\langle \overline{z}\alpha^{\#}\beta\psi^{\#}\overline{f^{\#}}, g\right\rangle = \left\langle \beta\overline{zg}\cdot\alpha^{\#}\psi^{\#}, f^{\#}\right\rangle = \left\langle \alpha^{\#}\psi^{\#}\cdot C_\beta g, f^{\#}\right\rangle = 0,\end{aligned}$$

since $\alpha^{\#}\psi^{\#}\cdot C_\beta g \in \alpha^{\#} H^2$ and $f^{\#} \in K_{\alpha^{\#}}$.

Note that this part of the proof did not use the assumption that $\alpha(0) = \beta(0) = 0$.

For the converse assume that $B_{\varphi}^{\alpha,\beta} = 0$, $\varphi \in L^2$. Let $\psi \in K_{\alpha\beta^{\#}}$ be such that $\varphi - \overline{\psi} \in H^2 + \overline{\alpha\beta^{\#} H^2}$. More precisely, $\psi = P_{\alpha\beta^{\#}}[\overline{(I-P)\varphi}]$. By the first part of the proof $B_{\varphi}^{\alpha,\beta} = B_{\overline{\psi}}^{\alpha,\beta} = 0$. Note that $\alpha(0)\beta^{\#}(0) = 0$, and so $\psi(0) = 0$. To complete the proof we show that $\psi = 0$. Since $C_\alpha k_0^\alpha = \frac{\alpha(z)}{z} \in K_\alpha$, we have

$$0 = B_{\overline{\psi}}^{\alpha,\beta} C_\alpha k_0^\alpha = P_\beta J\left(\overline{\psi}\cdot\frac{\alpha}{z}\right) = P_\beta\left(\psi^{\#}\overline{\alpha^{\#}}\right) = P_\beta J^{\#}\left(\psi\overline{\alpha}\right) = J^{\#} P_{\beta^{\#}}\left(\overline{\alpha}\psi\right).$$

Hence $P_{\beta^{\#}}\left(\overline{\alpha}\psi\right) = 0$, which means that $\overline{\alpha}\psi \perp K_{\beta^{\#}}$ and $\psi \perp \alpha K_{\beta^{\#}}$. Since $\psi \in K_{\alpha\beta^{\#}} = K_\alpha \oplus \alpha K_{\beta^{\#}}$, we get that $\psi \in K_\alpha$. It can be easily verified that $\left(B_{\overline{\psi}}^{\alpha,\beta}\right)^* = B_{\overline{\psi^{\#}}}^{\beta,\alpha}$. From this,

$$0 = B_{\overline{\psi^{\#}}}^{\beta,\alpha} k_0^\beta = P_\alpha J(\overline{\psi^{\#}}) = P_\alpha(\overline{z}\psi),$$

and ψ must be a constant function. But $\psi(0) = 0$, so $\psi \equiv 0$. □

Similarly to the proof of [10, Thm. 2.1], the proof of Theorem 2.1 will use the Crofoot transform. For an inner function α and $w \in \mathbb{D}$ the Crofoot transform is the multiplication operator J_w^α given by

$$J_w^\alpha f(z) = \frac{\sqrt{1-|w|^2}}{1-\overline{w}\alpha(z)} f(z). \tag{1}$$

The operator J_w^α is a unitary operator from K_α onto K_{α_w}, where

$$\alpha_w(z) = \frac{w-\alpha(z)}{1-\overline{w}\alpha(z)}. \tag{2}$$

Moreover,

$$\left(J_w^\alpha\right)^* = \left(J_w^\alpha\right)^{-1} = J_w^{\alpha_w}$$

(the details can be found in [8] or [11]).

It was proved in [2] that $B \in \mathcal{H}(\alpha)$ if and only if $J_w^\alpha B\Big(J_w^\alpha\Big)^{-1} \in \mathcal{H}(\alpha_w)$.

Lemma 2.3. *Let α, β be two inner functions. Let $a, b \in \mathbb{D}$ and let the functions α_a, β_b and the operators J_a^α, J_b^β be defined as in (2) and (1), respectively. If B is a bounded linear operator from K_α into K_β, then $B \in \mathcal{H}(\alpha, \beta)$ if and only if $J_b^\beta B\Big(J_a^\alpha\Big)^{-1} \in \mathcal{H}(\alpha_a, \beta_b)$. Moreover, if $B = B_\varphi^{\alpha,\beta}$, then $J_b^\beta B\Big(J_a^\alpha\Big)^{-1} = B_\phi^{\alpha_a,\beta_b}$ with*

$$\phi = \frac{(1-\bar{a}\alpha)(1-b\beta^{\#})}{\sqrt{1-|a|^2}\sqrt{1-|b|^2}} \cdot \varphi. \tag{3}$$

Proof. Assume first that $B = B_\varphi^{\alpha,\beta} \in \mathcal{H}(\alpha,\beta)$ with $\varphi \in L^2$. Then, for $f \in K_{\alpha_a}^\infty$, $g \in K_{\beta_b}^\infty$,

$$\begin{aligned}
\Big\langle J_b^\beta B_\varphi^{\alpha,\beta}\Big(J_a^\alpha\Big)^{-1} f, g\Big\rangle &= \Big\langle B_\varphi^{\alpha,\beta}\Big(J_a^\alpha\Big)^{-1} f, (J_b^\beta)^{-1} g\Big\rangle = \Big\langle B_\varphi^{\alpha,\beta} J_a^{\alpha_a} f, J_b^{\beta_b} g\Big\rangle \\
&= \Big\langle P_\beta J(I-P)\Big(\varphi \cdot J_a^{\alpha_a} f\Big), J_b^{\beta_b} g\Big\rangle = \Big\langle \varphi \cdot J_a^{\alpha_a} f, J J_b^{\beta_b} g\Big\rangle \\
&= \Big\langle \varphi \cdot J_a^{\alpha_a} f, J\Big(\tfrac{1-\bar{b}\beta}{\sqrt{1-|b|^2}} \cdot g\Big)\Big\rangle \\
&= \Big\langle \varphi \cdot J_a^{\alpha_a} f, \overline{J^{\#}\Big(\tfrac{1-\bar{b}\beta}{\sqrt{1-|b|^2}}\Big)} \cdot Jg\Big\rangle \\
&= \Big\langle \varphi \cdot J_a^{\alpha_a} f \cdot J^{\#}\Big(\tfrac{1-\bar{b}\beta}{\sqrt{1-|b|^2}}\Big), Jg\Big\rangle \\
&= \Big\langle \tfrac{1-\bar{a}\alpha}{\sqrt{1-|a|^2}} \cdot \tfrac{1-b\beta^{\#}}{\sqrt{1-|b|^2}} \cdot \varphi \cdot f, Jg\Big\rangle = \Big\langle B_\phi^{\alpha_a,\beta_b} f, g\Big\rangle,
\end{aligned}$$

where

$$\phi = \frac{(1-\bar{a}\alpha)(1-b\beta^{\#})}{\sqrt{1-|a|^2}\sqrt{1-|b|^2}} \cdot \varphi.$$

Hence

$$J_b^\beta B_\varphi^{\alpha,\beta}\Big(J_a^\alpha\Big)^{-1} = B_\phi^{\alpha_a,\beta_b},$$

with ϕ as in (3).

Assume now that B is a bounded linear operator from K_α into K_β such that

$$J_b^\beta B\Big(J_a^\alpha\Big)^{-1} = B_\phi^{\alpha_a,\beta_b} \in \mathcal{H}(\alpha_a, \beta_b).$$

From the first part of the proof and the fact that $(\alpha_a)_a = \alpha$, $(\beta_b)_b = \beta$,

$$B = J_b^{\beta_b}\Big[J_b^\beta B\Big(J_a^\alpha\Big)^{-1}\Big]\Big(J_a^{\alpha_a}\Big)^{-1} = B_\varphi^{\alpha,\beta},$$

with

$$\varphi = \frac{(1-\bar{a}\alpha_a)(1-b\beta_b^{\#})}{\sqrt{1-|a|^2}\sqrt{1-|b|^2}} \cdot \phi$$

(note that $\beta_b^\#$ denotes $J^\#\beta_b$). A simple calculation shows that

$$\phi = \frac{(1-\overline{a}\alpha)(1-b\beta^\#)}{\sqrt{1-|a|^2}\sqrt{1-|b|^2}} \cdot \varphi.$$

□

Proof of Theorem 2.1. The proof of the fact that $B_\varphi^{\alpha,\beta} = 0$ for $\varphi \in H^2 + \overline{\alpha\beta^\# H^2}$ was already given in the the first part of the proof of Proposition 2.2.

Assume now that $B_\varphi^{\alpha,\beta} = 0$ with $\varphi \in L^2$. If $\alpha(0) = \beta(0) = 0$, then $\varphi \in H^2 + \overline{\alpha\beta^\# H^2}$ by Proposition 2.2.

If $\alpha(0) \neq 0$ or $\beta(0) \neq 0$, then put $a = \alpha(0)$, $b = \beta(0)$ and define α_a, β_b as in (2). By Lemma 2.3,

$$0 = J_b^\beta B_\varphi^{\alpha,\beta}\left(J_a^\alpha\right)^{-1} = B_\phi^{\alpha_a,\beta_b},$$

where

$$\phi = \frac{(1-\overline{a}\alpha)(1-b\beta^\#)}{\sqrt{1-|a|^2}\sqrt{1-|b|^2}} \cdot \varphi.$$

Since $\alpha_a(0) = 0$ and $\beta_b(0) = 0$, Proposition 2.2 implies that $\phi \in H^2 + \overline{\alpha_a\beta_b^\# H^2}$ and

$$\phi = \frac{(1-\overline{a}\alpha)(1-b\beta^\#)}{\sqrt{1-|a|^2}\sqrt{1-|b|^2}} \cdot \varphi = h_1 + \overline{\alpha_a\beta_b^\# h_2}$$

for some $h_1, h_2 \in H^2$ (as before, $\beta_b^\# = J^\#\beta_b$). Hence, on the unit circle,

$$\begin{aligned}\varphi &= \tfrac{\sqrt{1-|a|^2}\sqrt{1-|b|^2}}{(1-\overline{a}\alpha)(1-b\beta^\#)} \cdot h_1 + \tfrac{\sqrt{1-|a|^2}\sqrt{1-|b|^2}}{(1-\overline{a}\alpha)(1-b\beta^\#)} \cdot \tfrac{\overline{a}-\overline{\alpha}}{1-a\overline{\alpha}} \cdot \tfrac{b-\overline{\beta^\#}}{1-\overline{b\beta^\#}} \cdot \overline{h_2} \\ &= \tfrac{\sqrt{1-|a|^2}\sqrt{1-|b|^2}}{(1-\overline{a}\alpha)(1-b\beta^\#)} \cdot h_1 + \overline{\alpha}\overline{\beta^\#} \cdot \overline{\tfrac{\sqrt{1-|a|^2}\sqrt{1-|b|^2}}{(1-\overline{a}\alpha)(1-b\beta^\#)} \cdot h_2} \in H^2 + \overline{\alpha\beta^\# H^2}.\end{aligned}$$

□

Corollary 2.4. *If $B \in \mathcal{H}(\alpha,\beta)$, then there exists $\psi \in K_{\alpha\beta^\#}$ such that $B = B_{\overline{\psi}}^{\alpha,\beta}$. If ψ is one such function, then the most general one is $\chi = \psi + c \cdot k_0^{\alpha\beta^\#}$, with c a scalar.*

Proof. Let $B \in \mathcal{H}(\alpha,\beta)$, $B = B_\varphi^{\alpha,\beta}$, $\varphi \in L^2$. By Theorem 2.1,

$$B_\varphi^{\alpha,\beta} = B_{\overline{\psi}}^{\alpha,\beta},$$

with $\psi = P_{\alpha\beta^\#}(\overline{\varphi})$. Since

$$\overline{k_0^{\alpha\beta^\#}} = 1 - \alpha(0)\beta^\#(0)\overline{\alpha\beta^\#} \in H^2 + \overline{\alpha\beta^\# H^2},$$

we clearly have

$$B_{\overline{\psi}}^{\alpha,\beta} = B_{\overline{\psi + ck_0^{\alpha\beta^\#}}}^{\alpha,\beta}$$

for each complex number c. Moreover, if $\psi, \chi \in K_{\alpha\beta^\#}$ and $B^{\alpha,\beta}_{\overline{\psi}} = B^{\alpha,\beta}_{\overline{\chi}}$, then $\overline{\psi - \chi} \in H^2 + \overline{\alpha\beta^\# H^2}$. Hence there are functions $h_1, h_2 \in H^2$ such that $\overline{\psi - \chi} = h_1 + \overline{\alpha\beta^\# h_2}$. From this h_1 must be a constant and

$$\psi - \chi = c + \alpha\beta^\# h_2$$

for some complex number c. Thus

$$\psi - \chi = P_{\alpha\beta^\#}(\psi - \chi) = P_{\alpha\beta^\#}(c + \alpha\beta^\# h_2) = c \cdot k_0^{\alpha\beta^\#}.$$

□

References

[1] R. V. Bessonov, *Fredholmness and compactness of truncated Toeplitz and Hankel operators*, Integr. Equ. Oper. Theory **82** no. 4 (2015), 451–467.

[2] C. Gu, *Algebraic properties of truncated Hankel operators*, preprint.

[3] C. Gu, D. Kang, *Rank of Truncated Toeplitz Operators*, Complex Anal. Oper. Theory **11** no. 4 (2017), 825–842.

[4] C. Câmara, J. Jurasik, K. Kliś-Garlicka, M. Ptak, *Characterizations of asymmetric truncated Toeplitz operators*, Banach J. Math. Anal. **11** no. 4 (2017), 899–922.

[5] M. C. Câmara, J. R. Partington, *Asymmetric truncated Toeplitz operators and Toeplitz operators with matrix symbol*, J. Operator Theory **77** no. 2 (2017), 455–479.

[6] M. C. Câmara, J. R. Partington, *Spectral properties of truncated Toeplitz operators by equivalence after extension*, J. Math. Anal. Appl. **433** no. 2 (2016), 762–784.

[7] J. A. Cima, S. R. Garcia, W. T. Ross, W. R. Wogen, *Truncated Toeplitz operators: spatial isomorphism, unitary equivalence, and similarity*, Indiana Univ. Math. J. **59** no. 2 (2010), 595–620.

[8] R. B. Crofoot, *Multipliers between invariant subspaces of the backward shift*, Pacific J. Math. **166** no. 2 (1994), 225–246.

[9] S. R. Garcia, W. T. Ross, *Recent progress on truncated Toeplitz operators*, in: J. Mashreghi, E. Fricain (eds.), Blaschke products and their applications, Fields Inst. Commun. **65**, Springer, New York, 2013, 275–319.

[10] J. Jurasik, B. Łanucha, *Asymmetric truncated Toeplitz operators equal to the zero operator*, Ann. Univ. Mariae Curie-Skłodowska, Sect. A **70** no. 2 (2016), 51–62.

[11] D. Sarason, *Algebraic properties of truncated Toeplitz operators*, Operators and Matrices **1** no. 4 (2007), 491–526.

Institute of Mathematics
Maria Curie-Skłodowska University
pl. M. Curie-Skłodowskiej 1, PL-20-031 Lublin
Poland
E-mail: bartosz.lanucha@poczta.umcs.lublin.pl
malgorzata.michalska@poczta.umcs.lublin.pl

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KIEDY ASYMETRYCZNY OBCIĘTY OPERATOR HANKELA RÓWNY JEST OPERATOROWI ZEROWEMU?

S t r e s z c z e n i e

W niniejszej pracy definiujemy klasę asymetrycznych obciętych operatorów Hankela. Następnie opisujemy symbole tych asymetrycznych obciętych operatorów Hankela, które równe są operatorowi zerowemu.

Słowa kluczowe: przestrzeń modelowa, obcięty operator Toeplitza, obcięty operator Hankela, asymetryczny obcięty operator Hankela