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NUMERICAL AND ANALYTICAL DETERMINATION OF COMPACT THERMAL MODEL PARAMETERS

Summary

The estimation of the heat transfer coefficient values for still air condition case is considered in this paper. The mentioned values are estimated using the analytical and numerical methods. The numerical solutions have been obtained using the Newton-Raphson algorithm, Levenberg-Marquardt algorithm and the conjugate gradient method. The obtained results have been precisely discussed. Moreover, comparison of the evaluation time, number of iterations and quality of solutions have been included. Apart from that, the accuracy of yielded findings is also demonstrated.

Keywords and phrases: Newton-Raphson algorithm, Levenberg-Marquardt algorithm, conjugent gradient method, average heat transfer coefficient, thermal problems, air conditions, compact thermal model

1. Introduction

The significant miniaturization of modern electronic structures and the meaningful growth of the frequency of their operation have big influence on the increase of the density of the power which is dissipated. Due to this fact the thermal management and the modelling of modern integrated circuits and electronic devices are currently one of the crucial issues in modern electronic designing, testing and using in daily life. The mentioned miniaturization is the response to customers expectations related to the utility, convenience, modernity and mobility of many useful electronic appliances. Apart from that, customers require reliable operation of the devices which they use.

However, electronic equipment brakes down from time to time. Research has shown that the major reason of that kind of appliances damages are caused by thermal problems [1], [2]. Comparison of the main reasons of microelectronic systems

malfunction is presented in Fig. 1. As it can be seen the most important factors which have the biggest influence on the improper operation of electronic appliances are temperature, vibrations, humidity and dust. It is clearly visible that the temperature rise induced the 55% of the failures of the electronic systems. It means that malfunctions caused by temperature reasons occur more often that the other failures caused by the all other factors mentioned previously. Due to this fact the proper estimation of the temperature rise in such kind of electronic devices is one of the most significant development steps in modern electronic designing. Apart from that such kind of analyses can be also useful for the modelling of heat distribution in investigated structure as well as for the estimation of device operation conditions. For example, they allow determining temperature dependences of the electronic appliances properties.

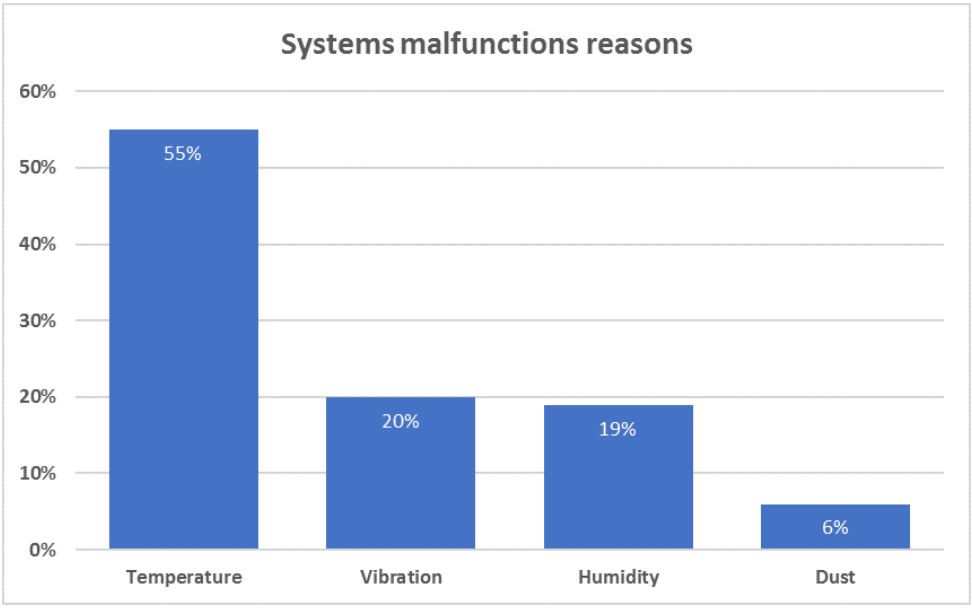

Fig. 1. *Reasons of microelectronic systems malfunctions.*

The parameter, which has the significant influence on the temperature distribution in the electronic structures is a heat transfer coefficient [3]. Determination of a value of the heat transfer coefficient is very complicated and complex problem due to the fact that the investigated coefficient value depends strongly on such factors as surface temperature or its geometry [4].

The temperature rises are based on the experimental data prepared using a diode made of silicon carbide. During the measurements, the mentioned diode was mounted

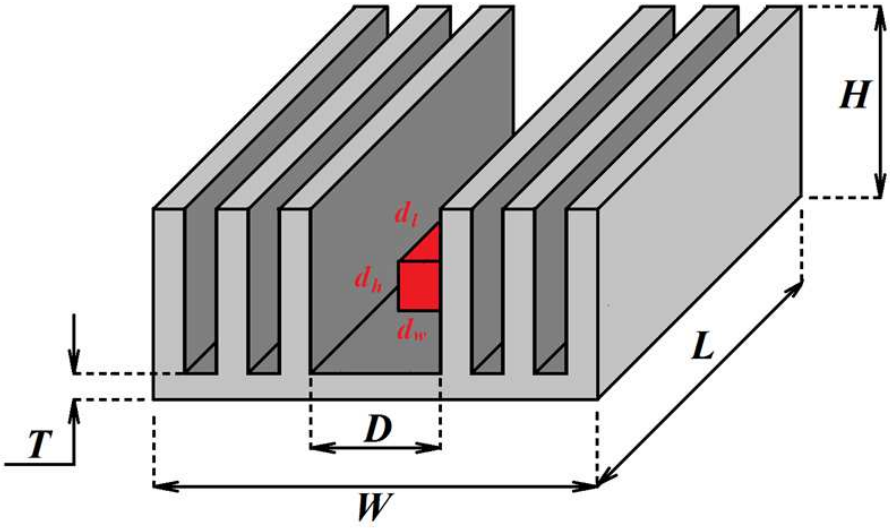

Fig. 2. *The cross-sectional view of the silicon carbide diode mounted in aluminum heat sink.*

on an U-shaped heat sink with thick aluminum plate. The measurements have been carried out in still air chamber. The schematic three-dimensional view of the described heat sink with silicon carbide diode is presented in Fig. 2.

All dimensions of the diode and heat sink have been precisely measured and their value are presented in Table 1. All measurements have been prepared in still air conditions. The device has been heated as long as the thermal steady state has been reached. After that, the temperature rises in the junction above the ambient temperature have been calculated.

The main task in this paper is to determine a mathematical model of the heat transfer coefficient which includes both the temperature rise values. The next sections of this paper include the following considerations. Firstly, the description of the analytical and numerical methods, which have been employed in order to determine the mathematical model describing the heat transfer coefficient values, are presented in detail. Among the numerical methods, the Newton-Raphson algorithm, the Levenberq-Marquardt algorithm and the conjugate gradient method have been chosen. Secondly, the description of the simulations is demonstrated. Finally, the simulation results are shown and carefully discussed.

Table 1. *Dimensions of the package.*

Parameter name	**Parameter symbol**	**Parameter value**
Heat sink length	L	25 mm
Heat sink width	W	29 mm
Heat sink height	H	12 mm
Heat sink area		$4350\,\mathrm{mm}^2$
Heat sink volume		$3500\,\mathrm{mm}^2$
Distance between fins in the middle part of the heat sink	D	12 mm
Thickness of the bottom part of the heat sink	T	2.5 mm
Diode maximum length	d_l	15 mm
Diode width	d_w	10 mm
Diode maximum height	d_h	4.5 mm

2. Methodology

In order to obtain the heat transfer coefficient values based on measurements described in the previous section, analytical and numerical methods have been proposed. Firstly, values of the heat transfer coefficient have been determined analytically and numerically based on measurements described in the previous section of this paper. Then, the dependences between the calculated heat transfer coefficient values and temperature and air velocity values will be found numerically. Moreover, the description of the chosen numerical methods will be demonstrated.

2.1. Analytical Determination of the Heat Transfer Coefficient

The first step in determination of the heat transfer coefficient value is to propose the proper thermal model based on measurements results and structure used during the measurement process. In this model two paths for the heat removing are considered. The first one contains a heat sink, a heat slug, a semiconductor die and a heat spreader. The generated heat is transferred into the ambient through the mentioned heat sink. The second one consists of a semiconductor die, slim layer of the air and a top part of the package. The proposed static thermal model is presented in Fig. 3 Explanation of the symbols presented in Fig. 3 are demonstrated in following Table 2.

In theoretical considerations, the heat removal paths containing the air layer can be neglected due to the fact that the approximated thermal resistance of the mentioned layer is huge. Moreover, the area of the surface of the top part of the

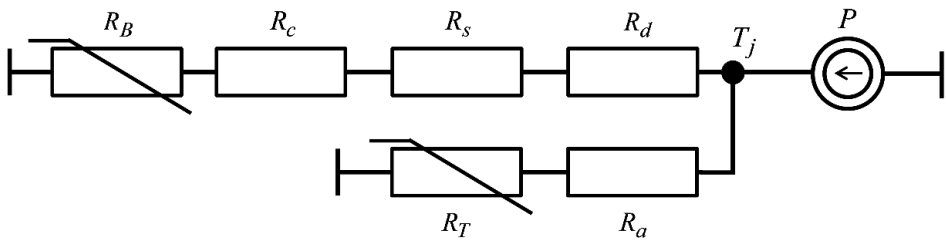

Fig. 3. *Equivalent RC thermal model.*

Table 2. *Parameters of equivalent RC thermal model.*

Parameter	**Interpretation**
R_T	Thermal resistances modelling the heat exchange with ambient
R_a	Thermal resistance of heat conduction through the lid of the package
R_d	Thermal resistance of the semiconductor die
R_S	Heat spreader resistance
R_c	Thermal resistance of the heat diffusion through the heat slug to the heat sink
R_B	Thermal resistances depend on the average heat transfer coefficient value and the area of the surface exchanging heat with ambient
T_j	Temperature rise in the junction where the heat is generated
P	Generated power

package is only a small percentage of the total surface area. However, in order to make the analysis more detailed, both heat removal paths are taken into consideration.

Values of the thermal resistances related to the heat slug, the heat spreader, the semiconductor die and the air are constant and they can be calculated according to formula 1.

$$R = \frac{d}{k \cdot S}. \tag{1}$$

Symbols in the equation above have the meanings explained in Table 3.
However, the thermal resistances of the top and the bottom part of the package depend on the value of the average heat transfer coefficient, so they vary for different values of the mentioned coefficient. The mentioned thermal resistance values can be estimated using the following dependence (2)

Table 3. *Parameters of thermal resistance formula.*

Parameter	Interpretation
R	Thermal resistances
S	Surface area
d	Thickness of the analyzed layer
k	Thermal condactivity of the material

$$R = \frac{1}{h \cdot S}. \tag{2}$$

The parameter h reflects the value of the heat transfer coefficient. Considering equation 2 and proposed static thermal model, it is possible to obtain the value of the thermal resistance in junction R_j using the formula presented below:

$$R_j = \frac{(R_a + R_T(h_{average})) \cdot (R_d + R_S + R_c + R_B(h_{average}))}{(R_a + R_T(h_{average})) + (R_d + R_S + R_c + R_B(h_{average}))}. \tag{3}$$

Parameter $h_{average}$ means the average heat transfer coefficient.

Assuming that S_T, S_B are respectively the top and the bottom surface area of the package and that the constants A, B, C are described by expressions 4, 5, 6:

$$A = (S_T \cdot S_B)^{-1}, \tag{4}$$

$$B = (S_T)^{-1} \cdot (R_d + R_S + R_c - R_j) + (S_B)^{-1} \cdot (R_a - R_j), \tag{5}$$

$$C = R_a \cdot (R_d + R_S + R_c) - R_j \cdot (R_a + R_d + R_S + R_c). \tag{6}$$

and doing some algebraic transformations, the value of the average heat transfer coefficient can be obtained analytically directly from equation 7 as its positive root.

$$A \cdot \left(\frac{1}{h_{average}}\right)^2 + B \cdot \left(\frac{1}{h_{average}}\right) + C = 0. \tag{7}$$

2.2. The Newton-Raphson Method

The Newton-Raphson method is numerical method which finds approximations of the roots of the real-valued functions [5], [6]. Each successive approximation is better than that one which has been found in the previous iteration. It means that in each successive iteration it reflects the coordinates of the zero of a function more accurately. In the classical Newton-Raphson method the following assumptions are needed:

- the function f and its first and second derivatives, $f^{'}$ and $f^{''}$ respectively, are continuous inside the interval $[a, b]$, $a, b \in \mathbb{R}$, $a < b$,
- the function f has exactly one root inside the interval $[a, b]$,
- the starting point belongs to the interval $[a, b]$,
- the value of the first derivative of the analyzed function f in the starting point is not equal to zero,
- the first and the second derivatives of the analyzed function f do not change their signs inside the interval $[a, b]$,
- values of the analyzed function f in the left and in the right end of the interval $[a, b]$ have different signs, what means that $f(a) \cdot f(b) < 0$.

The iterative formula used in the Newton-Raphson method is presented below:

$$\begin{cases} x_0 = p, & p \in \mathbb{R} \\ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, & n \in \mathbb{N} \cup \{0\}. \end{cases} \tag{8}$$

In order to obtain the simulation results, related to the heat transfer coefficient values determination, an objective function f has to be defined. Thus, formulas describing the equivalent thermal resistances RE_1 and RE_2 have been proposed and presented below:

$$RE_1\left(h_{average}\right) = \frac{1}{h_{average} \cdot S_T} + \frac{d_a}{k_a \cdot S_a}, \tag{9}$$

$$RE_2\left(h_{average}\right) = \frac{d_d}{k_d \cdot S_d} + \frac{d_s}{k_s \cdot S_s} + \frac{d_c}{k_c \cdot S_c}\frac{1}{h_{average} \cdot S_B}. \tag{10}$$

New symbols, which have been used in equations above, have the meanings presented in Table 4.

Then, the objective function f depended on the value of the variable h has been formulated according to the following expression:

$$f\left(h_{average}\right) = \left(\Delta T - \frac{RE_1\left(h_{average}\right) \cdot RE_2\left(h_{average}\right)}{RE_1\left(h_{average}\right) + RE_2\left(h_{average}\right)} \cdot P\right)^2. \tag{11}$$

After that, the iterative Newton-Raphson formula can be used in order to heat transfer coefficient determination. However, the simulation results will be presented in the next chapter.

2.3. The Levenberg-Marquardt Method

The second numerical method, which can be helpful in the heat transfer coefficient determination, is the Levenberq-Marquardt method [5], [6]. The presented algorithm is called *the Damped Least-Squares method*. This method is usually employed to obtain the solutions of non-linear problems, especially of the least squares problems.

Table 4. *Equivalent thermal resistance parameters interpretation.*

Parameter	**Interpretation**
d_a	Thicknes of the air layer
k_a	Thermal conductivity of the air
S_a	Surface of the air layer
d_d	Thicknes of the semiconductor die
k_d	Thermal conductivity of the semiconductor die
S_d	Surface of the semiconductor die
d_s	Thicknes of the heat spreader
k_s	Thermal conductivity of the heat spreader
S_s	Surface of the heat spreader
d_c	Thicknes of the heat slug
k_c	Thermal conductivity of the heat slug
S_c	Surface of the heat slug

Moreover, the investigated Levenberq-Marquardt algorithm can be also used for fitting of the least squares curves. The mentioned algorithm can find the local minimum only. The valuable feature of the Levenberq-Marquardt algorithm is its robustness. It is reflected in solution finding even in the case of the ill-conditioned problems.

It is assumed that the set of the empirical data (X, Y) is known. Both X and Y are the vectors consisting of n elements, $n \in \mathbb{N} \setminus \{0\}$. Thus, $X = \{x_1, \ldots, x_n\}$ and $Y = \{y_1, \ldots, y_n\}$. The main task is to find the fitting $\hat{y} = f\left(x \mid (p_1, \ldots, p_m)\right)$, $m \in \mathbb{N}\setminus\{0\}$. It is also assumed that the best fitting minimizes the following functional:

$$fit\left(p_1, \ldots, p_m\right) = \sum_{i=1}^{n} \left(y_i - f\left(x_i \mid (p_1, \ldots, p_m)\right)\right)^2. \tag{12}$$

In general case, the Levenberq-Marquard algorithm can find the solution of nonlinear optimization problem. Thus, it can be written in the following form:

$$F\left(s_1, \ldots, s_n\right) = \frac{1}{2} \sum_{i=1}^{m} d_i^2\left(s_1, \ldots, s_n\right), \quad m \leq n. \tag{13}$$

Assuming that $s = (s_1, \ldots, s_n)$, the functional F can be minimized using the formula presented below:

$$s_{n+1} \approx s_n - \left(\nabla^2 F\left(s_n\right) + \frac{1}{\lambda} \cdot I\right)^{-1} \cdot \left(\nabla F\left(s_n\right)\right), \quad n \in \mathbb{N} \setminus \{0\} \tag{14}$$

The parameter λ, $\lambda > 0$, is called *the damping factor* and it is adjusted in every iteration. The symbol I means the identity matrix. In the case when the parameter λ is large, the problem can be approximated using the following quadratic form:

$$s_{n+1} \approx s_n - \left(\nabla^2 F\left(s_n\right)\right)^{-1} \cdot \left(\nabla F\left(s_n\right)\right). \tag{15}$$

On the other hand, in the case when the value of the parameter λ is close to zero, the problem of the minimization of the functional F can be solved using the steepest descent method. The mentioned method can be described according to the following formula:

$$s_{n+1} \approx s_n - \lambda \cdot \left(\nabla F\left(s_n\right)\right). \tag{16}$$

For the heat transfer coefficient determination problem, the presented previously function f, which has to be fitted, is formulated as follows:

$$f\left(h_{average}\right) = \frac{\left(\frac{1}{h_{average} \cdot S_T} + \frac{d_a}{k_a \cdot S_a}\right) \cdot \left(\frac{d_d}{k_d \cdot S_d} + \frac{d_s}{k_s \cdot S_s} + \frac{d_c}{k_c \cdot S_c} + \frac{1}{h_{average} \cdot S_B}\right)}{\left(\frac{1}{h_{average} \cdot S_T} + \frac{d_a}{k_a \cdot S_a}\right) + \left(\frac{d_d}{k_d \cdot S_d} + \frac{d_s}{k_s \cdot S_s} + \frac{d_c}{k_c \cdot S_c} + \frac{1}{h_{average} \cdot S_B}\right)} \cdot P. \tag{17}$$

Similarly, to the previous algorithm, the simulation results related to determination of heat transfer coefficient value will be demonstrated in the next chapter.

2.4. The Conjugate Gradient Method

The conjugate gradient algorithm is the numerical method for solving some kinds of linear equations systems [5], [6]. The investigated method allows finding solutions of these systems of equations whose matrices are positively defined and symmetric. The conjugate gradient method is the iterative method and it can be used to find solution of the optimization problem.

The description of the conjugate method can be presented in the following way. Firstly, the proper system of linear equations has to be defined. It can be formulated in the form presented below:

$$Mx = b. \tag{18}$$

The symbol M represents the matrix of n rows and n columns which is positively defined, symmetric and real-valued. The vector $x = [x_1, \ldots, x_n]^T$ is the vector of variables and symbol T means transposition. On the other hand, $b = [b_1, \cdot, b_n]^T$ is the vector of intercepts. The solution of the presented system of equations is expressed by $\hat{x}$. Secondly, the conjugate vectors p_k, where k denotes the number of iteration, have to be selected. These vectors will be helpful in approximation of the solution of systems of equations $\hat{x}$. Then the start point x_{start} is chosen. In order to make

the analyses easier, it is assumed that $x_{start} = ([0, \ldots, 0]_{1\times n})^T$. The solution of $\hat{x}$ minimizes the quadratic form presented below:

$$F(x) = \frac{1}{2}x^T Mx - b^T x. \tag{19}$$

After that, the residual in k^{th} step res_k is defined as:

$$res_k = b - Mx_k. \tag{20}$$

It leads to formulate the expression described the conjugate vectors p in the next iteration:

$$p_k = res_k - \sum_{i<k} \frac{p_i^T M res_i}{p_i^T M p_i} p_i. \tag{21}$$

It allows approximating the optimal location in the next iteration. This location can be described using the formula below:

$$x_{k+1} = x_k + \lambda_k p_k, \tag{22}$$

where

$$\lambda_k = \frac{p_i^T res_{k-1}}{p_i^T M p_k}. \tag{23}$$

In order to determine the heat transfer coefficient values, the presented conjugate gradient method has to be adjusted. Firstly, a vector of measured values of the temperature rise above the ambient temperature has to be constructed. Its construction is presented in equation (24):

$$Y = [Y_1, \ldots, Y_n]^T. \tag{24}$$

Parameter m means the number of measurements and Y_i, $i \in \{1, \ldots, n\}$, is the temperature rise measured in the i^{th} measurement. Then, a vector of unknown parameters has been defined:

$$P = [h_1, \ldots, h_m]^T, \quad n \geq m. \tag{25}$$

Considering the problem presented in this paper, it can be assumed that $n = m$, because number of unknown parameters is related to the number of measurements. Moreover, only one value of the heat transfer coefficient is appropriate to the one measurement. Thus, in further analysis, the vector P will be equivalently written as $h = [h_1, \ldots, h_n]^T$. Therefore, the objective function can be written in the following form:

$$S(h) = \sum_{i=1}^{n} [Y_i - T_i(h_i)]^T. \tag{26}$$

Vector $T(h) = [T(h_1), \ldots, T(h_n)]^T$ is the vector of estimated temperatures depended on the values of the unknown parameters. The next step is related to the sensitivity matrix $J(h)$ determination according to the following expression:

$$J(h) = \left[\frac{\partial T^T(h)}{\partial h}\right]^T = \begin{bmatrix} \frac{\partial T_1}{\partial h_1} & \frac{\partial T_1}{\partial h_2} & \frac{\partial T_1}{\partial h_3} & \cdots & \frac{\partial T_1}{\partial h_n} \\ \frac{\partial T_2}{\partial h_1} & \frac{\partial T_2}{\partial h_2} & \frac{\partial T_2}{\partial h_3} & \cdots & \frac{\partial T_2}{\partial h_n} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial T_n}{\partial h_1} & \frac{\partial T_n}{\partial h_2} & \frac{\partial T_n}{\partial h_3} & \cdots & \frac{\partial T_n}{\partial h_n} \end{bmatrix}. \tag{27}$$

Thus, the gradient of the objective function S is calculated and the possible local minimum is found as follows:

$$-2J^T(h)[Y - T(h)] = 0. \tag{28}$$

Therefore, the gradient direction $\nabla S(h)$ is obtained using the formula presented below:

$$\nabla S(h) = -2J^T(h)[Y - T(h)]. \tag{29}$$

It means that the j^{th} component of the gradient direction can be determined as follows:

$$[\nabla S(h)]_j = -2\sum_{i=1}^{n} \frac{\partial T_i}{\partial h_j}[Y_i - T_i(h_i)], \quad j = 1, \ldots, n. \tag{30}$$

The conjugate coefficients in k^{th} iteration γ^k are computed using the Polak-Ribiere iterative formula presented below:

$$\begin{cases} \gamma^0 = 0, & k = 0 \\ \gamma^k = \frac{\sum_{j=1}^{n}\left[\left[\nabla S\left(h^k\right)\right]_j\left[\nabla S\left(h^k\right) - \nabla S\left(h^{k-1}\right)\right]_j\right]}{\sum_{j=1}^{n}[\nabla S(h^{k-1})]_j^2}, & k \in \mathbb{N} \setminus \{0\} \end{cases}. \tag{31}$$

Thus, the direction of the descent in k^{th} iteration d^k can be obtained using the following expression:

$$d^k = \nabla S\left(h^k\right) + \gamma^k d^{k-1}, \quad k \in \mathbb{N} \setminus \{0\}, \tag{32}$$

where d^0 is the gradient of the objective function calculated using equation (29). Moreover, the step size in k^{th} iteration β^k has to be also computed:

$$\beta^k = \frac{\left[J^k d^k\right]^T \left[T\left(h^k\right) - Y\right]}{\left[h^k d^k\right]^T \left[h^k d^k\right]}, \quad k \in \mathbb{N} \setminus \{0\}. \tag{33}$$

Then, the new estimation is calculated as follows:

$$h^{k+1} = h^k - \beta^k d^k, \quad k \in \mathbb{N} \setminus \{0\}. \tag{34}$$

The algorithm is stopped after fixed number of iterations or when the demanded accuracy is obtained. The main simulation results will be demonstrated in the next section.

3. Simulations and Results

This section includes description of the simulation process. Moreover, all simulations results are presented in detail. All simulations are carried out in Matlab environment. The computational unit used during the simulations contains the four-cores and eight-threads Intel®Core™ i7 CPU (2.5 GHz nominally, 3.5 GHz using turbo mode), 16 GB RAM DDR3 memory and Microsoft Windows operating system. Apart from that, all the algorithms use the sparse matrices implementation.

3.1. Simulation Description

During simulations the parameters values shown in Table 5 have been used. Moreover, the measured values of the temperature of the junction T_j, temperature rise in the junction $\triangle T$ and the generated power P have been taken into consideration. These values are demonstrated in Table 6. Simulations results are presented in the next subsections.

Table 5. *Parameters values used during simulations.*

Parameter	**Value/Expression**	**Parameter**	**Value/Expression**
S_T	$100.0000\,\mathrm{mm}^2$	S_B	$4350.0000\,\mathrm{mm}^2$
R_T	$R_T = \frac{1}{h \cdot S_T}$	R_B	$R_B = \frac{1}{h \cdot S_B}$
d_a	$4.0000\,\mathrm{mm}$	d_s	$0.2600\,\mathrm{mm}$
k_a	$0.1000\,\frac{\mathrm{W}}{\mathrm{m \cdot K}}$	k_s	$140.0000\,\frac{\mathrm{W}}{\mathrm{m \cdot K}}$
S_a	$5.1529\,\mathrm{mm}^2$	S_s	$5.1529\,\mathrm{mm}^2$
R_a	$77626.0000\,\frac{\mathrm{K}}{\mathrm{W}}$	R_s	$0.3604\,\frac{\mathrm{K}}{\mathrm{W}}$
d_d	$0.3865\,\mathrm{mm}$	d_c	$1.1250\,\mathrm{mm}$
k_d	$220.0000\,\frac{\mathrm{W}}{\mathrm{m \cdot K}}$	k_c	$240.0000\,\frac{\mathrm{W}}{\mathrm{m \cdot K}}$
S_d	$5.1529\,\mathrm{mm}^2$	S_c	$5.1529\,\mathrm{mm}^2$
R_d	$0.3409\,\frac{\mathrm{K}}{\mathrm{W}}$	R_c	$0.9097\,\frac{\mathrm{K}}{\mathrm{W}}$

Table 6. *Measured values of the temperature and the generated power.*

Number of measurement	**Generated Power** $P\,[W]$	**Junction temp.** $T_j\,[K]$	**Junction temp. rise** $\triangle T\,[K]$
1.	0.099	23.3448	3.5448
2.	0.204	26.1331	6.3331
3.	0.312	29.0844	9.2844
4.	0.425	31.4849	11.6849
5.	0.542	34.2885	14.4885
6.	0.662	36.9479	17.1479
7.	0.786	40.0547	20.2547
8.	0.916	42.5716	22.7716
9.	1.049	45.8152	26.0152
10.	1.187	47.7529	27.9529
11.	1.331	50.3522	30.5522
12.	1.481	53.0508	33.2508
13.	1.635	55.7636	35.9636
14.	1.797	58.6007	38.8007
15.	1.967	61.9732	42.1732
16.	2.142	65.2134	45.4134
17.	2.327	68.4293	48.6293
18.	2.519	71.4793	51.6793
19.	2.717	74.1120	54.3120
20.	2.935	79.1753	59.3753

3.2. Simulation Results

All results demonstrated in this chapter have been obtained based on the parameters values presented in subsection 3.1. The heat transfer coefficient values have been obtained analytically using the method described in subsection 2.1. The simulation results are plotted in Figure 4.

On the other hand, based on the Newton-Raphson method description presented in subsection 2.2, and using the parameters values demonstrated in subsection 3.1, the results plotted in Figure 5 have been yielded. Moreover, the start point $h_0 = 5$ has been chosen. Then, the Levenberg-Marquardt method has been used in order to receive the heat transfer coefficient values. The starting point h_0 has been fixed and its values is equal to 5. Considering the Levenberg-Marquardt method, presented in subsection 2.3, the results demonstrated in Figure 6 have been received. Finally, the simulation results, related to the conjugate gradient method, have been obtained.

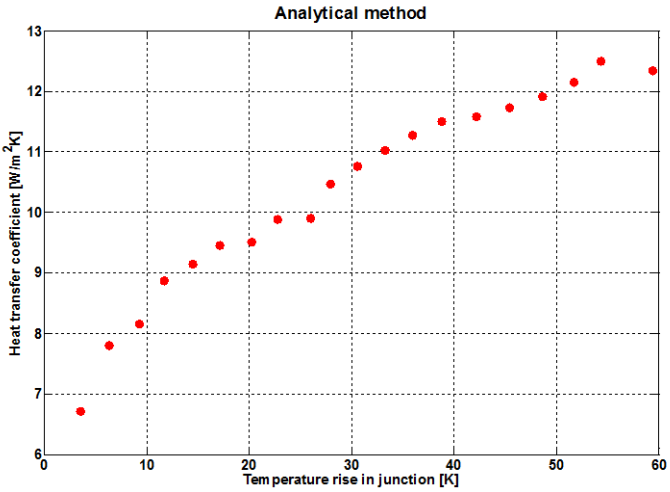

Fig. 4. *Heat transfer coefficient values obtained for the analytical method.*

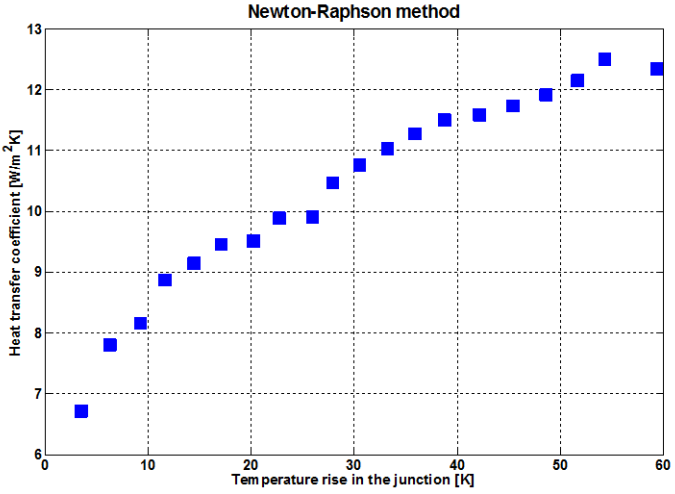

Fig. 5. *Heat transfer coefficient values obtained for the Newton-Raphson method.*

They are demonstrated in Figure 7. Accordingly to the previous cases, the start value of the heat transfer coefficient h_0 has been equated to 5. Table 7 includes the heat transfer coefficient values determined using all investigated methods.

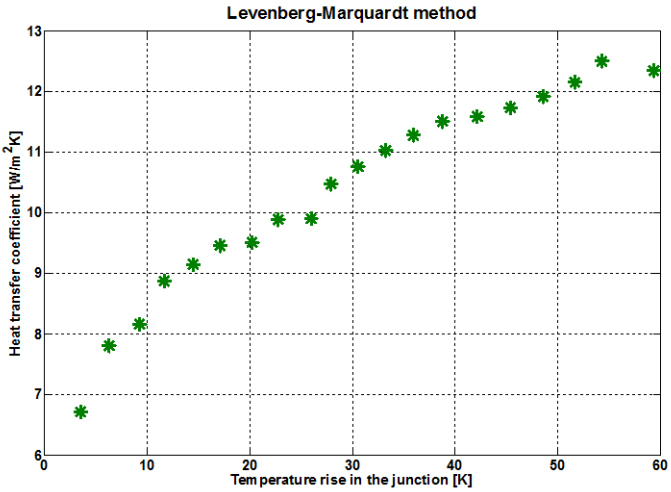

Fig. 6. *Heat transfer coefficient values obtained for the Levenberg-Marquardt method.*

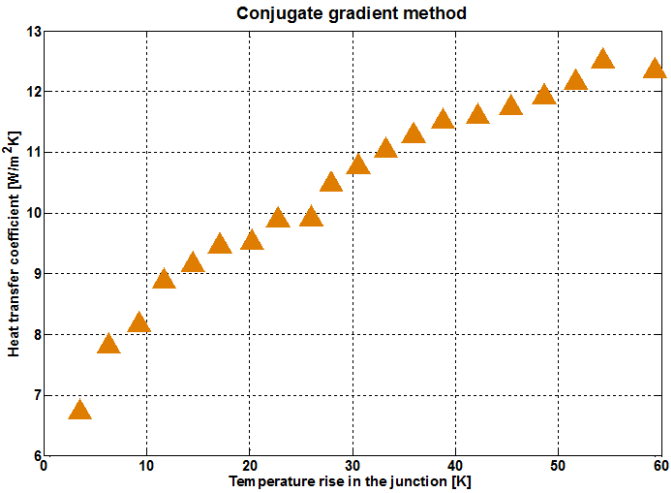

Fig. 7. *Heat transfer coefficient values obtained for the conjugate gradient method.*

As it can be seen, the results obtained using all the mentioned methods are similar. Moreover, results yielded using employed numerical methods coincide with these ones which have been calculated analytically. Apart from that, the differences between the heat transfer coefficient values obtained using the direct and numerical formulas are irrelevant. This situation indicates that all the presented numerical methods are

Table 7. *Heat transfer coefficient values comparison.*

Num. of meas.	**Junc. temp. rise** $[K]$	**Estimated heat transfer coefficient values** $\left[\frac{W}{m^2 \cdot K}\right]$			
		Analytical method	**Newton--Raphson**	**Levenberg--Marquardt**	**Conjugate gradient**
1.	3.5448	6.719518	6.719117	6.719518	6.718535
2.	6.3331	7.807035	7.806283	7.807035	7.806434
3.	9.2844	8.164184	8.164184	8.164184	8.164093
4.	11.6849	8.878489	8.878472	8.878489	8.878522
5.	14.4885	9.147993	9.147828	9.147993	9.148051
6.	17.1479	9.460037	9.460037	9.460037	9.459941
7.	20.2547	9.512457	9.512457	9.512457	9.512477
8.	22.7716	9.884705	9.884705	9.884705	9.884692
9.	26.0152	9.910199	9.910199	9.910199	9.910183
10.	27.9529	10.475383	10.475382	10.475383	10.475385
11.	30.5522	10.767441	10.767441	10.767441	10.767432
12.	33.2508	11.027259	11.027259	11.027259	11.027283
13.	35.9636	11.273734	11.273734	11.273734	11.273701
14.	38.8007	11.501820	11.501776	11.501820	11.501882
15.	42.1732	11.589751	11.589750	11.589751	11.589784
16.	45.4134	11.731151	11.731145	11.731151	11.731144
17.	48.6293	11.915853	11.915853	11.915853	11.915854
18.	51.6793	12.156714	12.156714	12.156714	12.156734
19.	54.3120	12.504787	12.504787	12.504787	12.504764
20.	59.3753	12.343296	12.343286	12.343296	12.343307

convergent and they can be used to heat transfer coefficient determination. It is very important, especially in very big and complex heat transfer problems which cannot be solved analytically.

It is also worth analysing which algorithm demands the lowest computational power or which one estimates the heat transfer coefficient values in the shortest time. Table 8 includes information about the number of iterations and times spent for heat transfer coefficient values evaluations using each of the numerical algorithms presented in this paper. As it can be seen, number of iterations is not related with the time spent for heat transfer coefficient values determination. It is a result of the algorithms specifications. The conjugate gradient method demands the lowest number of iterations, however each loop contains a big number of multiplication and summation

Table 8. *Evaluation times and number of iterations of the algorithms.*

Number of measurement	**Number of iterations of the algorithm**		
	Newton--Raphson	**Levenberg--Marquardt**	**Conjugate gradient**
1.	4	4	
2.	3	4	
3.	3	5	
4.	3	5	
5.	2	5	
6.	3	5	
7.	2	5	
8.	3	5	
9.	2	5	
10.	3	5	
11.	3	5	
12.	3	5	
13.	3	5	
14.	2	6	
15.	2	6	
16.	2	6	
17.	2	6	
18.	3	6	
19.	3	6	
20.	2	6	
Sum	53	105	37
Evaluated time [s]	5.838789	0.501296	22.451791

operations. Moreover, due to the fact that all the parameters are evaluated at the same time, it is not possible to calculate the number of iterations demanded for evaluation each of them separately. On the other hand, the biggest number of iterations has been received for the Levenberg-Marquardt method. However, the heat transfer coefficient values have been calculated in the shortest time, because the smallest number of summations and multiplications has been needed. It can be stated that for the investigated heat transfer problem the Levenberg-Marquardt algorithm is characterized by the lowest computational complexity among the presented numerical algorithms.

4. Conclusions

The analytical and numerical approaches to determination of the heat transfer coefficient are presented in this paper. Comparing the solutions obtained using different numerical algorithms, it can be stated that the best method is the Levenberq-Marquardt algorithm. The mentioned algorithm is characterized by the shortest evaluation time.

The estimation of the heat transfer coefficient is very important because its value has a big influence on the temperature of every electronic device. This issue is also significant for development of the newest electronic integrated circuits. Precise determination of the temperature distribution in these structures is crucial for the proper operation of the entire electronic systems. Moreover, it can help in further optimization and can reduce a number of failures caused by thermal problems.

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NUMERYCZNE I ANALITYCZNE WYZNACZANIE PARAMETRÓW KOMPAKTOWYCH MODELI TERMICZNYCH

S t r e s z c z e n i e

Niniejszy artykuł dotyczy estymacji wartości współczynnika wymiany ciepła w przypadku naturalnych, niewymuszonych warunków chłodzenia. Wartości ww. współczynnika szacowane są z wykorzystaniem metod analitycznych i numerycznych. Numeryczne rozwiązanie opisywanego problemu zostało otrzymane przy użyciu algorytmu Newtona-Raphsona, algorytmu Levenberga-Marquardta i metody gradientu sprzężonego. Uzyskane wyniki zostały szczegółowo omówione w artykule. Ponadto, artykuł zawiera również porównanie czasu, liczby iteracji i dokładności poszczególnych serii wyników.

Słowa kluczowe: algorytm Newtona-Raphsona, algorytm Levenberga-Marquardta, metoda gradientu sprzężonego, średnia wartość współczynnika wymiany ciepła, problemy termiczne, warunki chłodzenia, kompaktowy model termiczny