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**BINARY AND TERNARY STRUCTURES IN PHYSICS III GALOIS-TYPE THEORY FOR BINARY AND TERNARY STRUCTURES**

**Summary**

A Galois-type theory for Turing machine is presented as well as its counterpart for binary and ternary structures in physics. In addition an idea of binary-ternary decomposition of quinary and senary structures is indicated with application to polymer research.

*Keywords and phrases:* noncommutative Galois extensions, finite-dimensional algebras, associative rings and algebras, binary physical structure, ternary physical structure, quinary physical structure, senary physical structure

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1. Galois extension theory for Turing machine
2. Generation of an $m$-words system and its Galois extension
3. The noncommutative Galois-type theory for elementary particles
4. Galois-type theory for Kobayashi-Masukawa model
5. The description of the 3-generations
6. A counterpart for alloys and polymers

This paper is a continuation of parts I and II of the paper: *Binary and ternary structures in physics*. Part I: *The hierarchy structure of Turing machine in physics*, by Osamu Suzuki [19] and Part II: *Binary and ternary structures in elementary particles physics vs. those in physics of condensed matter*, by Osamu Suzuki, Julian Ławrynowicz, and Agnieszka Niemczynowicz [20].

# 1. Galois extension theory for Turing machine

In this Section we introduce the concept of noncommutative Galois-type theory and construct the theory for the Turing machine of $m$-words.

## Galois-type theory for algebras

We introduce the theory of a (noncommutative) Galois-type theory for an algebra. Let $\mathcal{A}$ be an algebra and let $\Theta_1, \Theta_2, \ldots, \Theta_N$ be a system of complex matrices. We make the extension of $\mathcal{A}$ by $\Theta_i$ $(i = 1, 2, \ldots, N)$ in the following way:

$$\mathcal{A}[\Theta] = \{a_1\Theta_1 + a_2\Theta_2 + \cdots + a_N\Theta_N \mid a_i \in \mathcal{A}\},$$

where $a_i\Theta_i = a_i \otimes \Theta_i$, $(i = 1, 2, \ldots, N)$. We put

$$\Theta = a_1\Theta_1 + a_2\Theta_2 + \cdots + a_N\Theta_N.$$

The authomorphism

$$G : \ \Theta \to \Theta',$$

where $\Theta' = a_1\Theta'_1 + a_2\Theta'_2 + \cdots + a_N\Theta'_N$ is called *Galois group action* if $G|_{\mathcal{A}}$ =identity. When $G^2$ =Id or $G^3$ =Id holds, the extension is called *binary* (resp. *ternary*). The typical Galois group is generated by the shift operation:

$$G : \ \Theta_i \to \Theta_{i+1}.$$

When the extension consists of two parts $\{\Theta_i\}$ and $\{\overline{\Theta}_i\}$ which are called *conjugate*, we have the conjugation relation

$$\mathcal{A}[\Theta, \overline{\Theta}] = \{\Sigma a_i\Theta_i,\ \Sigma a_i\overline{\Theta}_i\}$$

and we have the conjugation operation: $\Theta_i \to \overline{\Theta}_i$ $(\overline{\Theta}_i \to \Theta_i)$.

## Galois-type theory for Turing machine

We introduce a concept of Galois extension for Turing machine $\mathcal{L}^{(m)}$ whose invariant elements are identical with $\{a_1^n a_2^n \ldots a_m^n \mid n > 0\}$.

(1) The Galois extension of $\mathcal{L}^{(2)}$

We consider $\mathcal{L}^{(2)}$ which is generated by $a_1$ and $a_2$

$$\mathcal{L}^{(2)} = \{a_1^n a_2^n \mid n > 0\}.$$

We construct a Galois extension with the invariance $\mathcal{L}^{(2)}$. Putting

$$\begin{cases} \Theta = a_1\Theta_1 + a_2\Theta_2 \\ \bar{\Theta} = a_1\bar{\Theta}_1 + a_2\bar{\Theta}_2 \end{cases}$$

we shall make the Galois $\mathcal{A}[\Theta, \bar{\Theta}]$ extension over $\mathcal{A}[a_1, a_2]$. We take the Galois group which is generated by the conjugation

$$g: \ \Theta \to \bar{\Theta}.$$

Then we can see that the extension is binary. Next we proceed to the construction of the extension explicitly:

$$\begin{aligned}\Theta\bar{\Theta} &= (a_1\Theta_1 + a_2\Theta_2)(a_1\bar{\Theta}_1 + a_2\bar{\Theta}_2)\\ &= a_1^2\Theta_1\bar{\Theta}_1 + a_1a_2(\Theta_1\bar{\Theta}_2 + \Theta_2\bar{\Theta}_1) + a_2^2\Theta_2\bar{\Theta}_2.\end{aligned}$$

Hence choosing

$$\Theta_1\bar{\Theta}_1 = 0, \quad \Theta_2\bar{\Theta}_2 = 0, \quad \Theta_1\bar{\Theta}_2 + \Theta_2\bar{\Theta}_1 = 1$$

we can obtain

$$\Theta\bar{\Theta} = a_1a_2 \otimes 1.$$

When we choose

$$\Theta_1 = \bar{\Theta}_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \Theta_2 = \bar{\Theta}_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

the above condition is satisfied.

Replacing $a_1 \implies a_1^n$, $a_2 \implies a_2^n$, we can obtain the desired invariant space.

(2) The Galois extension of $\mathcal{L}^{(3)}$

We construct the Galois extension for $\mathcal{L}^{(3)}$. We shall make the extension over the elements:

$$\mathcal{L}^{(3)} = \{a_1^n a_2^n a_3^n \mid n > 0\}.$$

We consider

$$\begin{aligned}\Theta &= a_1\Theta_1 + a_2\Theta_2 + a_3\Theta_3,\\ \Theta' &= a_1\Theta_2 + a_2\Theta_3 + a_3\Theta_1,\\ \Theta'' &= a_1\Theta_3 + a_2\Theta_1 + a_3\Theta_2.\end{aligned}$$

We consider the Galois group which is generated by the shift operation:

$$g: \ \Theta \to \Theta', \quad \Theta' \to \Theta'', \quad \Theta'' \to \Theta.$$

Next we determine the $\{\Theta_i\}$. Putting

$$\Theta_i\Theta_j\Theta_k = \begin{cases} I_3 & i, j, k \text{ are different} \\ 0 & \text{otherwise.} \end{cases}$$

Then we have

$$\Theta\Theta'\Theta'' = a_1a_2a_3 \otimes I_3.$$

Hence we obtain the desired Galois extension. The Galois-type theory for $\mathcal{L}^{(m)}$ can be obtained in an analogous manner.

The 3-generation in Galois extension

Next we proceed to the 3-generation of $m$-words system. We will describe the mechanism in terms of the Galois extension. We take

$$\mathcal{L}^{(4)} = \{a_1^n a_2^n a_3^n a_4^n \mid n > 0\}.$$

Then, following the process as described for $\mathcal{L}^{(3)}$, in this case we have:

$$\begin{cases} \Theta & = & a_1\Theta_1 + a_2\Theta_2 + a_3\Theta_3 + a_4\Theta_4 \\ G(\Theta) & = & a_2\Theta_1 + a_3\Theta_2 + a_4\Theta_3 + a_1\Theta_4 \\ G^2(\Theta) & = & a_3\Theta_1 + a_4\Theta_2 + a_1\Theta_3 + a_2\Theta_4 \\ G^3(\Theta) & = & a_4\Theta_1 + a_1\Theta_2 + a_2\Theta_3 + a_3\Theta_4. \end{cases}$$

We consider the invariant form:

$$H = \Theta \cdot G(\Theta) \cdot G^2(\Theta) \cdot G^3(\Theta).$$

We can introduce the algebraic structure by

$$\Theta_i\Theta_j\Theta_k\Theta_l = \begin{cases} I_4 & if\ i, j, k, l \text{ are different,} \\ 0 & \text{otherwise.} \end{cases}$$

Then we have

$$H = a_1a_2a_3a_4 \otimes I_3.$$

Next we proceed to the 3-generations of the algebra. We consider

$$\mathcal{A}[\Theta_1, \Theta_2, \Theta_3, \Theta_4] = \mathcal{A}[\Theta_4]\mathcal{A}[\Theta_1, \Theta_2, \Theta_3],$$

i.e. we treat the algebra of $\Theta_1, \Theta_2, \Theta_3$ with the parameter algebra $\mathcal{A}[\Theta_4]$. we treat the Galois extension structure

$$\Theta_1 \stackrel{G}{\to} \Theta_2 \stackrel{G}{\to} \Theta_3 \stackrel{G^2}{\to} \Theta_1$$

"ignoring the parameter space", we can consider the ternary Galois extension. Then we have the invariant form

$$H' = Y(a_4)a_1a_2a_3.$$

Making a different decomposition:

$$\mathcal{A}[\Theta_1, \Theta_2, \Theta_3, \Theta_4] = \mathcal{A}[\Theta_1]\mathcal{A}[\Theta_2, \Theta_3, \Theta_4]$$

we can obtain

$$H'' = X(a_1)a_2a_3a_4.$$

Hence we have

$$H = H' \cap H''.$$

## 2. Generation of an $m$-words system and its Galois extension

In physics and biology we can observe several types of generations of $m$-words systems. Here we give several generations and try to construct their systems.

**Generations of $m$-words systems**

Here we give several classes of generations of $m$-words systems. For the description we prepare a "match sticks" and, by making combinations of them we generate systems.

(1) Binary system

In this case we have only one type of generation

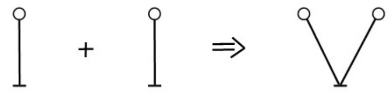


Fig. 1. Generation for binary system.

(2) Ternary system

In this case we have the following two types:
**$\alpha$-spontaneous generation**

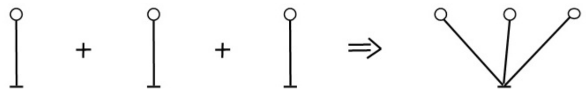


Fig. 2. $\alpha$ -spontaneous generation of ternary system.

**$\beta$-successive generation**

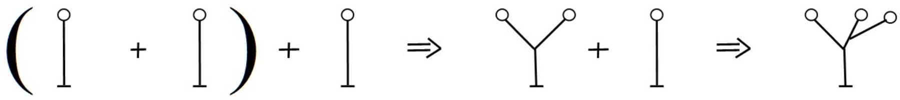


Fig. 3. $\beta$-successive generation of ternary system.

We prefer to write this generation in the following manner:

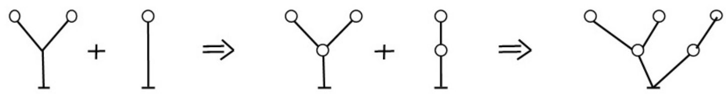


Fig. 4. Two-step generation of ternary system.

*Examples*

**(1) Spontaneous generation**

($\alpha$) colors of quarks

($\beta$) $x, y, z$ of the space dimensions

**(2) Successive generation**

($\alpha$) Generation of curbon by 3-Heliums

($\beta$) Transcription mechanism in molecular biology

### The Galois extensions for generations

We have given the Galois extension for the spontaneous generation. Below we give the Galois extensions for successive generations.

At first we make the binary extension which we have given in Section 1:

$$\hat{\mathcal{L}}^{(2)} = \left\{ a_1\Theta_1 + a_2\Theta_2 : \; a_1, a_2 \in \mathcal{L}^{(2)} \right\}.$$

Next we will make a central extension of $\hat{\mathcal{L}}^{(2)}$:

$$\hat{\hat{\mathcal{L}}}^{(2)} = \hat{\mathcal{L}}^{(2)} \otimes \Theta_3\mathbb{R},$$

where $\Theta\Theta_3 = \Theta_3\Theta_i$, $i = 1, 2$. Then we have a ternary extension which corresponds to the implication

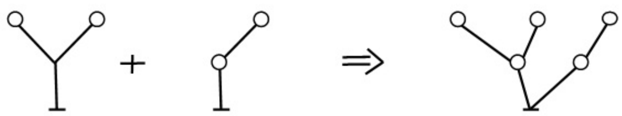


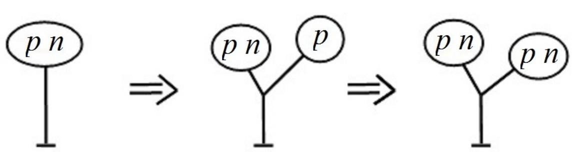
Fig. 5. Ternary extension.

$$\hat{\mathcal{L}}^{(2)} \oplus \Theta_3\mathbb{R} \implies \hat{\hat{\mathcal{L}}}^{(2)}$$

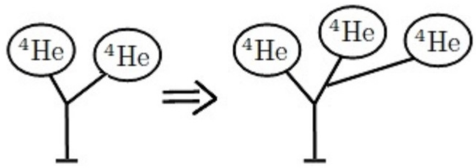
### Galois-type theory for atom physics

We apply the construction of Galois-type theory to atom physics which is given in Section 1.

Following the scheme of the construction of the binary Galois extension, we can obtain the following Galois extension:

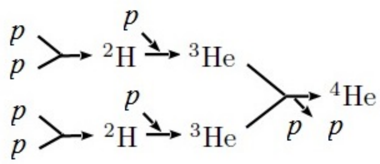


Following the scheme of the construction of the ternary Galois extension, we can obtain the following [7, 8, 12, 16, 20, 23]:

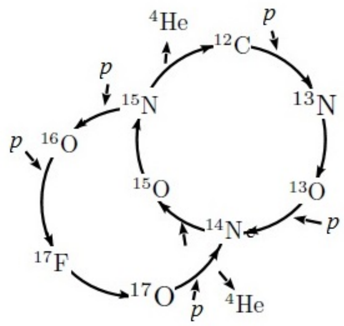



Here we notice the two types of generations of atoms

proton-proton chain




CNO cycle




From our observations of atoms by Galois extension we can propose the following problem:

**Problem.** Can we describe the generation of atoms in terms of the repetition of Galois extensions of binary and ternary type and can we obtain the Mendeleev table of atoms?

**Remark.** From the fact that the binary and ternary extension structures are restructured, we can determine the Galois extensions and we can describe binary and ternary structures.

We have the following theorems (c.f. [19]; a detailed proof will appear in *Fractals and chaos related to Ising-Onsager lattices. Quaternary approach vs ternary approach* by the present authors and A. Niemczynowicz, submitted for publication):

**Theorem I. (1)** *The nonion algebra is a ternary Galois extension of the algebra* $\mathbb{B} : \mathbb{N} = \mathbb{B}[\sqrt[3]{I_3}]$. *The extension can be realized by* $\mathbb{B}[\tau]$ $(\tau^3 = I_3)$ *with the choice of* $\tau = Q_i, \bar{Q}_i$ $(i = 1, 2, 3)$.

**(2)** $\widetilde{\mathbb{N}}$ *is a binary extension of* $\mathbb{B}' : \widetilde{\mathbb{N}} = \mathbb{B}'[\sqrt[2]{I_3}]$. *Hence we have the following* *commutative diagram:*

$$\begin{tikzcd} & \widetilde{\mathbb{N}} \arrow[dl, "T_4"'] \arrow[dr, "Q_i \ or \ \bar{Q}_j"] & \\ \mathbb{N} \arrow[dr, "Q_i \ or \ \bar{Q}_j"'] & & \mathbb{B}' \arrow[dl, "T_4"] \\ & \mathbb{B} & \end{tikzcd} \quad (i,j = 1,2,3) \tag{1}$$

**Theorem II. (3)** *We have the following Galois extensions:*

$$\begin{cases} \mathcal{A}[R] & = & \{xR_1 + yR_2 + zR_3 \,|\, x, y, z \in R[j]\}, \\ \mathcal{A}[Q_i] & = & \{xR_1 + yQ_i + z\bar{Q}_i \,|\, x, y, z \in R[j]\} \, (i = 1, 2, 3), \\ \mathcal{A}[\bar{Q}_i] & = & \{xR_1 + y\bar{Q}_i + zQ_i \,|\, x, y, z \in R[j]\} \, (i = 1, 2, 3). \end{cases} \tag{2}$$

*The extension does not depend on the choice of $\tau$ with $\mathcal{B}'[\tau]$ $(\tau^3 = 1)$. Namely we have*

$$\mathcal{N} = \mathcal{A}[Q_1] = \mathcal{A}[Q_2] = \mathcal{A}[Q_3]. \tag{3}$$

## 3. The noncommutative Galois-type theory for elementary particles

In this section we give a method of Galois-type theory for the description of the hierarchy structure in elementary particles. We shall be concerned with the following topics (c.f. [8, 12, 16, 23, 24]):

1. Nishijima and Gell-Mann formula,
2. Galois-type theory for Gell-Mann and Zwick model,
3. Galois-type theory for Kobayashi-Masukawa model.

### (1) Nishijima and Gell-Mann formula

In the well-known $SU(3)$ description for the quark theory for the three quarks: u, d, s the basic idea can be described as follows: Gell-Mann and Zwick gave the coordinates: ($\mathrm{I}_z$, Q, S), where $\mathrm{I}_z$ is the $z$-component of isospin, S: strangeness, Y: hyper charges, Q: charge.

Table 1. Gell-Mann and Zwick coordinates of quarks.

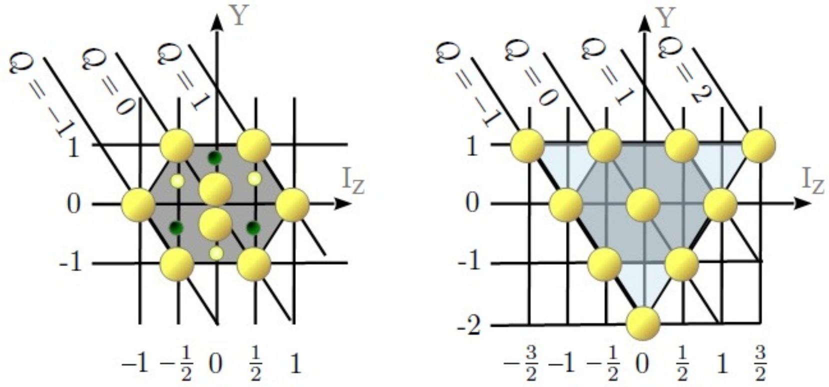
| Quark - Property | I | $I_z$ | S | B | Y | Q |
|---|---|---|---|---|---|---|
| u | 1/2 | 1/2 | 0 | 1/3 | 1/3 | 2/3 |
| d | 1/2 | -1/2 | 0 | 1/3 | 1/3 | -1/3 |
| s | 0 | 0 | - 1 | 1/3 | 0 | -1/3 |

By the use of the fundamental formula

$$Q = I_z + Y/2$$

we can plot their coordinates on the plane as follows:

Table 2. Another visualization of Gell-Mann and Zwick coordinates of quarks.




**(2) SU(3) Theory**

We can realize these configurations by the use of the representations of $SU(3)$. We consider $V^3(\mathbb{C})$ and $V_3(\mathbb{C})$:

$$V^3(\mathbb{C}) = \{(\xi^i)|\ i = 1,2,3\}, \quad V_3(\mathbb{C}) = \{(\xi^j)|\ j = 1,2,3\}.$$

Then we have the basic representations:

$$U:\ V^3(\mathbb{C}) \to V^3(\mathbb{C}), \quad (\xi^j) \implies (\xi'^{(j')}) = U(\xi^{(j)}),$$

$$U^*:\ V_3^*(\mathbb{C}) \to V_3^*(\mathbb{C}), \quad (\xi_j)^* \implies (\xi'_{j'}) = \xi_{j'}U^*.$$

By the use of the representation we construct the representation which describes mesons and baryons.

(i) Mesons

We consider the matrix

$$M_{ij} = \left(\xi \overset{i}{\otimes} \xi_j\right) \in M_3(\mathbb{C}).$$

Then we have the adjoint representation:

$$\mathrm{Ad}_V:\ \left(\xi \overset{i}{\otimes} \xi_j\right) \implies (U(\xi^i \xi_j)U^*)$$

which is denoted by $3^* \otimes 3$. The invariant space is given by

$$\begin{pmatrix} \frac{x^0}{\sqrt{2}} + \frac{y}{\sqrt{6}} & x^4 & K+ \\ -\pi^- & -\frac{x^0}{\sqrt{2}} + \frac{y}{\sqrt{6}} & K^0 \\ K^- & -K^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix},$$

by which we can obtain the following configuration which is identical with the Nishijima and Gell-Mann configuration:

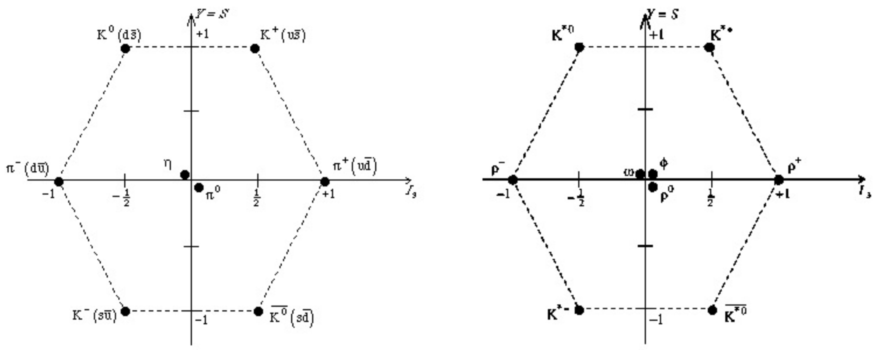



Fig. 6. The Nishijima and Gell-Mann configuration for mesons.

(ii) Baryons

In the same manner, putting

$$T^{ijk} = \xi^i \otimes \xi^j \otimes \xi^k$$

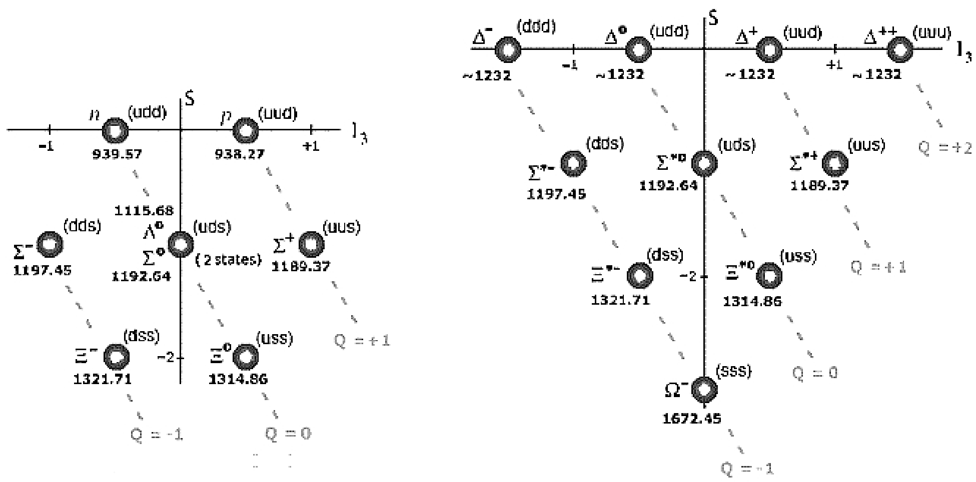
and considering representation

$$\mathrm{Ad}_U T = 3 \otimes 3 \otimes 3$$

we can obtain the following decomposition:

$$T = 1 \oplus 8 \oplus 8 \oplus 10.$$

Then we can associate baryons to 8 and 10 components.




Isospin 1/2, J = 1/2 Isospin 3/2, J = 3/2

Fig. 7. The Nishijima and Gell-Mann configuration for baryons.

These configurations are identical with the Nishijima and Gell-Mann configuration.

**(3) Noncommutative Galois theory for $SU(3)$ model**

Considering the fact that $SU(3)$ is not a continuous group but a discrete subgroup, we can realize the above model in terms of

$$A_3 \subset S_3 \subset SU(3),$$

where $S_3$ is the permutation group of three words and $A_3$ is an alternating subgroup of $S_3$.

(i) The construction of mesons by $A_3$

We consider an algebra which is generated by u, s, d:

$$\mathcal{A} = \mathcal{A}[u, s, d].$$

We make a binary extension $\mathcal{A}[u, s, d]$. We express it by:

$$\theta = u\theta_1 + d\theta_2 + s\theta_3, \quad \bar{\theta} = \bar{u}\bar{\theta_1} + \bar{d}\bar{\theta_2} + \bar{s}\bar{\theta_3}.$$

We find the invariance under the Galois group:

$$\theta \implies \bar{\theta}.$$

The invariant forms are

$$\begin{cases} H_1 = (u\theta_1 + d\theta_2 + s\theta_3)(\bar{u}\bar{\theta_1} + \bar{d}\bar{\theta_2} + \bar{s}\bar{\theta_3}) \\ H_0 = u_1\bar{u_1} + s\bar{s} + u\bar{u}. \end{cases}$$

Hence we have the matrix form of the invariant

$$H_1 = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ \bar{u}d & d\bar{d} & s\bar{d} \\ \bar{u}s & \bar{d}s & s\bar{s} \end{pmatrix},$$

$$H_2 = (u\bar{u} + d\bar{d} + s\bar{s}) \otimes 1_3.$$

Making

$$\widetilde{H}_1 = H_1 + \lambda H_2$$

we have the invariance

$$\widetilde{H}_1 = \begin{pmatrix} (1+\lambda)u\bar{u} + \lambda d\bar{d} + \lambda s\bar{s} & u\bar{d} & u\bar{s} \\ \bar{u}d & u\bar{u} + (1+\lambda)d\bar{d} + \lambda s\bar{s} & d\bar{s} \\ \bar{u}s & \bar{d}s & \lambda u\bar{u} + \lambda d\bar{d} + (1+\lambda)s\bar{s} \end{pmatrix}.$$

When we choose $\lambda = -\frac{1}{3}$, we can obtain the well known Ge'llMann's matrix:

$$\begin{pmatrix} \frac{1}{3}(2u\bar{u} - d\bar{d} - s\bar{s}) & u\bar{d} & u\bar{s} \\ \bar{u}d & \frac{1}{3}(-u\bar{u} + 2d\bar{d} - s\bar{s}) & d\bar{s} \\ \bar{u}s & \bar{d}s & \frac{1}{3}(-u\bar{u} - d\bar{d} + 2s\bar{s}) \end{pmatrix}.$$

**Remark.** We can also derive noncommutative Galois extension from the Dirac equation of $SU(3)$.

The construction of baryons by $S_3$

In a similar manner we can realize baryons by the use of ternary Galois extension. For this we take the algebra $\mathcal{A}[u, d, s]$ and make the ternary Galois extension:

$$\widetilde{\mathcal{A}}[u, d, s] = \{\theta_1 + G\theta_2 + G^2\theta_3 \mid \theta_i \in \mathcal{A}[u, d, s]\},$$

where

$$G = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Then we have the invariance:

$$(\theta_1 u + \theta_2 d + \theta_3 s)(\theta_1 u + \theta_2 uG^2 + \theta_3 sG)j(\theta_1 u + \theta_2 dG + \theta_3 sG^2)$$

$$= (uuu\theta_1^3 + ddd\theta_2^3 + sss\theta_3^3) \otimes 1$$

$$+[(uud + udu + duu)\theta_1^2\theta_2 + (ddu + dud + udd)\theta_2\theta_1^2 + (ssd + sds + dss)\theta_2\theta_3^2$$

$$+(dds + dsd + sdd)\theta_3^2\theta_2 + (uus + usu + suu)\theta_1^2\theta_3 + (ssu + sus + uss)\theta_1\theta_1^2$$

$$+(usd + uds + dus + dsu + sud + uds)\theta_1\theta_2\theta_3] \otimes \hat{G},$$

where

$$\hat{G} = 1 + G + G^2 \quad (\neq 0).$$

Hence we can obtain 10 (resp. 7) baryons from the symmetric elements when they satisfy the Pauli condition with (resp. without) colour condition (as for colours see Sect. 5 below).

## 4. Galois-type theory for Kobayashi-Masukawa model

Next we proceed to the construction of the noncommutative Galois-type theory for 3-generations of quarks. People say that the quarks t and b are created at first. Then the quark b and the anti-quark $\bar{\mathrm{b}}$ decay and the quark d and its conjugate $\bar{\mathrm{d}}$, and u,

$\bar{\mathrm{u}}$ are created:

$$\mathrm{e}^+ + \mathrm{e}^- \to \upsilon(4\mathrm{s})(=\mathrm{b}\bar{\mathrm{b}}) \to \left\{ \begin{array}{ccccc} \mathrm{B}^0 & + & \bar{\mathrm{B}}^0 & \to & \cdots \\ (\mathrm{d}\bar{\mathrm{b}}) & & (\mathrm{b}\bar{\mathrm{d}}) & & \cdots \\ & & & & \\ \mathrm{B}^+ & + & \bar{\mathrm{B}}^- & \to & \cdots \\ (\mathrm{u}\bar{\mathrm{b}}) & & (\bar{\mathrm{u}}\mathrm{b}) & & \cdots \end{array} \right.$$

By this process we can obtain (u,d) from (b). In a similar manner

$$\mathrm{e}^+ + \mathrm{e}^- \to \begin{array}{c} J/\Psi \\ (\mathrm{c}\bar{\mathrm{c}}) \end{array} \to \left\{ \begin{array}{ccc} \pi^+ & + & \pi^- \\ (\mathrm{u}\bar{\mathrm{d}}) & & (\bar{\mathrm{u}}\mathrm{d}) \\ & & \\ \mathrm{D}^+ & + & \bar{\mathrm{D}}^- \\ (\mathrm{c}\bar{\mathrm{d}}) & & (\bar{\mathrm{c}}\mathrm{d}) \end{array} \right.$$

$$\mathrm{e}^+ + \mathrm{e}^- \to \begin{array}{c} \Phi \\ (\mathrm{s}\bar{\mathrm{s}}) \end{array} \to \left\{ \begin{array}{ccccc} \mathrm{K}^+ & + & \mathrm{K}^- & & \\ (\mathrm{u}\bar{\mathrm{s}}) & & (\bar{\mathrm{u}}\mathrm{s}) & & \\ & & & & \\ \pi^+ & + & \pi^0 & + & \pi^- \\ (\mathrm{u}\bar{\mathrm{d}}) & & (\mathrm{d}\bar{\mathrm{d}}) & & (\mathrm{u}\bar{\mathrm{d}}) \end{array} \right.$$

By this process we can obtain (u,d), (c,d) form (c) and (u,s), (u,d) from (s). From these processes we have the generations by

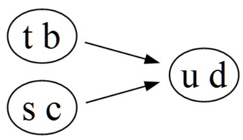



Fig. 8. Generation of quarks.

We shall try to describe the process in terms of Galois extension. We consider the algebra

$$\mathcal{A} = \mathcal{A}[\mathrm{u}, \mathrm{d}, \bar{\mathrm{u}}, \bar{\mathrm{d}}]$$

and the ternary Galois extension

$$\mathcal{A}(G) = \{\theta_1 + u\theta_2 + d\theta_3\} + \{\theta_1 + uG\theta_2 + dG^2\theta_3\} + \{\theta_1 + uG^2\theta_2 + dG\theta_3\}.$$

Here we understand the construction of quark generations as in the following diagrams:

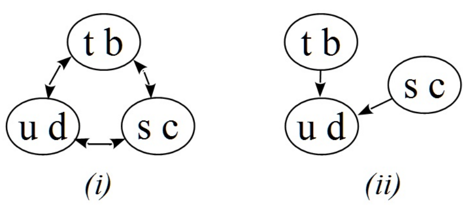



Fig. 9. Construction of quark generations vs. ternary Galois extension.

When each generation is equivalent to $(i)$, we can realize it in terms of the Galois extension. Putting

$$\begin{cases} \mathrm{uG} = \mathrm{c} & \mathrm{dG}^2 = \mathrm{s} \\ \mathrm{uG}^2 = \mathrm{s}, & \mathrm{uG} = t \end{cases}$$

we may describe the 3-generations of quark families.

In the case of $(ii)$ we may construct it in terms of the repetition of the binary extensions.

## 5. The description of the 3-generations

Next we proceed to the description on the origin of the 3-generations of the quarks. Here we shall make use of the fact:

$$\mathbb{Z}_2 \times \mathbb{Z}_3 = \mathbb{Z}_3 \times \mathbb{Z}_2 = \mathbb{Z}_6$$

and we assume the Gell-Mann model. Then we may accept three quarks: u, d, s. Here we assume that we have a binary extension. Then we can obtain three quarks: t, b, c. Hence we have the basic form.

$$H = \theta_1 u + \theta_2 d + \theta_3 s + \theta_4 c + \theta_5 b + \theta_6 t$$

and its conjugate form

$$H = \theta_1 \bar{u} + \theta_2 \bar{d} + \theta_3 \bar{s} + \theta_4 \bar{c} + \theta_5 \bar{b} + \theta_6 \bar{t}.$$

By this we can obtain the meson table $H$:

Table 2. The meson table

$$\Omega = \begin{pmatrix} \mathrm{u\bar{u}} & \mathrm{u\bar{d}} & \mathrm{u\bar{s}} & \mathrm{u\bar{c}} & \mathrm{u\bar{b}} & \epsilon\mathrm{u\bar{t}} \\ \mathrm{d\bar{u}} & \mathrm{d\bar{d}} & \mathrm{d\bar{s}} & \mathrm{d\bar{c}} & \mathrm{d\bar{b}} & \epsilon\mathrm{d\bar{t}} \\ \mathrm{s\bar{u}} & \mathrm{s\bar{d}} & \mathrm{s\bar{s}} & \mathrm{s\bar{c}} & \mathrm{s\bar{b}} & \epsilon\mathrm{s\bar{t}} \\ \mathrm{c\bar{u}} & \mathrm{c\bar{d}} & \mathrm{c\bar{s}} & \mathrm{c\bar{c}} & \mathrm{c\bar{b}} & \epsilon\mathrm{c\bar{t}} \\ \mathrm{b\bar{u}} & \mathrm{b\bar{d}} & \mathrm{b\bar{s}} & \mathrm{b\bar{c}} & \mathrm{b\bar{b}} & \epsilon\mathrm{d\bar{t}} \\ \epsilon\mathrm{t\bar{u}} & \epsilon\mathrm{t\bar{d}} & \epsilon\mathrm{t\bar{s}} & \epsilon\mathrm{t\bar{c}} & \epsilon\mathrm{t\bar{b}} & \epsilon^2\mathrm{t\bar{t}} \end{pmatrix}$$

Also we can obtain the baryon table which is generated by

$$\theta_1 u + \theta_2 d + \theta_3 s + \theta_4 c.$$

We choose the following triples:

[u,d,s], [u,d,c], [d,s,c], [u,s,c].

For each triple we make the ternary invariant form:

$$(\theta_1 u + \theta_2 d + \theta_3 s)(\theta_1 u + \theta_2 dG^2 + \theta_3 sG)(\theta_1 u + \theta_2 dG + \theta_3 sG^2)$$

and the others. We can obtain the baryon tables.

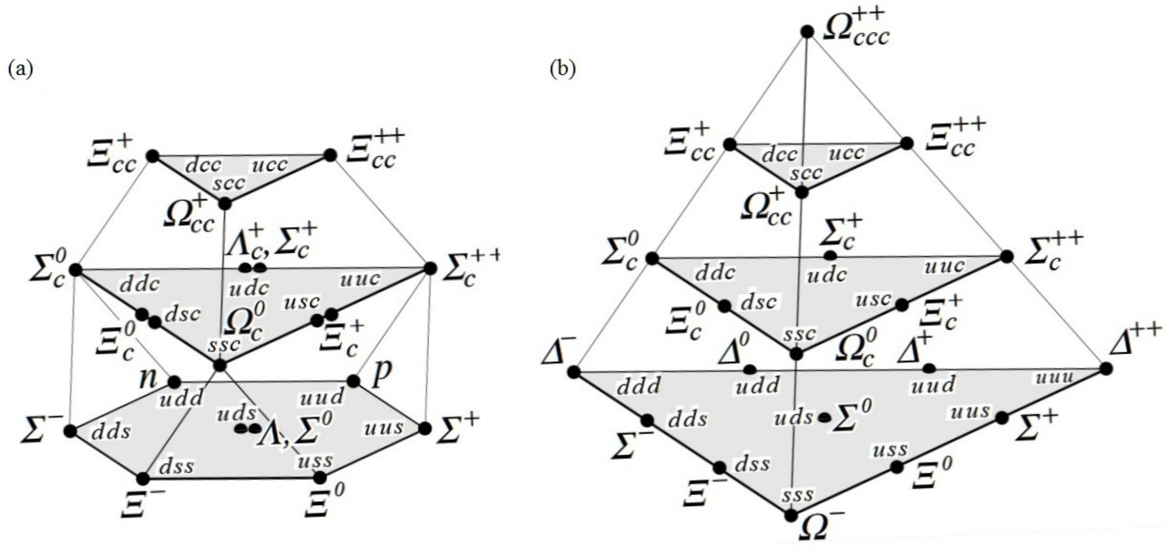



Fig. 10. The baryon tables (a) with and (b) without colour condition.

When we want to take the colour condition into account, we have to make a ternary Galois extension [13].

## 6. A counterpart for alloys and polymers

The above described theory has its counterpart for metallic alloys [12, 13, 14]. In this direction one can prove that [20, 19]

**Theorem III.** *We have the binary and ternary extension structures on* SU(3):
(**4**) *We have the following adjoint representation on* $L_i$ $(i = 1, 2, 3)$:

$$\begin{cases} He_1H^{-1} = -e_2, & He_2H^{-1} = e_1, & He_3H^{-1} = e_3, \\ H'e_1'H'^{-1} = -e_2', & H'e_2'H'^{-1} = -e_1', & H'e_3'H'^{-1} = e_3', \\ H'e_1''H'^{-1} = e_2'', & H'e_2''H'^{-1} = e_1'', & H'e_3''H'^{-1} = e_3'', \end{cases} \tag{4}$$

*where*

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad H' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}. \tag{5}$$

(**5**) *We can obtain the following commutation relations*:

$$\begin{cases} e_1^2 = e_2^2 = e_3^2 = -1 \\ e_1e_2 = -e_2e_1 = e_3, \; e_2e_3 = -e_3e_2 = e_1, \; e_3e_1 = -e_1e_3 = e_2. \end{cases} \tag{6}$$

*After the central extension, we have the Clifford algebra which is isomorphic to the quaternion algebra. For the case of* $e_i'$ *and* $e_i''$ $(i = 1, 2, 3)$, *we have the same assertions on* $L_i$ $(i = 1, 2, 3)$. *Hence we obtain the Dirac operators desired.*

(**6**) *We have*

$$G_1 e_k^{(0)} G_1^{-1} = e_k^{(1)} (k = 1,2,3),\ G_1 e_k^{(1)} G_1^{-1} = e_k^{(2)} (k = 1,2),\ G_1 e_3^{(1)} G_1^{-1} = -e_3^{(2)}$$
$$G_1 e_k^{(2)} G_1^{-1} = e_k^{(0)} (k = 1,3),\ G_1 e_2^{(2)} G_1^{-1} = -e_2^{(0)}, \tag{7}$$

*where*

$$G_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{8}$$

*After the central extension, we have the Clifford algebra which is isomorphic to the quaternion algebra. For the case of $e_i'$ and $e_i''$ $(i = 1,2,3)$, we have the same assertions on $L_i$ $(i = 1,2,3)$. Hence we obtain the Dirac operators desired.*

As far as polymers e.g. pentacene are concerned the idea of decomposing their structure to two: 2 Carbons-2 Hydrogens elements (Fig. 11) allows reducing relationships between senary-quinary structures to the ternary-binary structures.

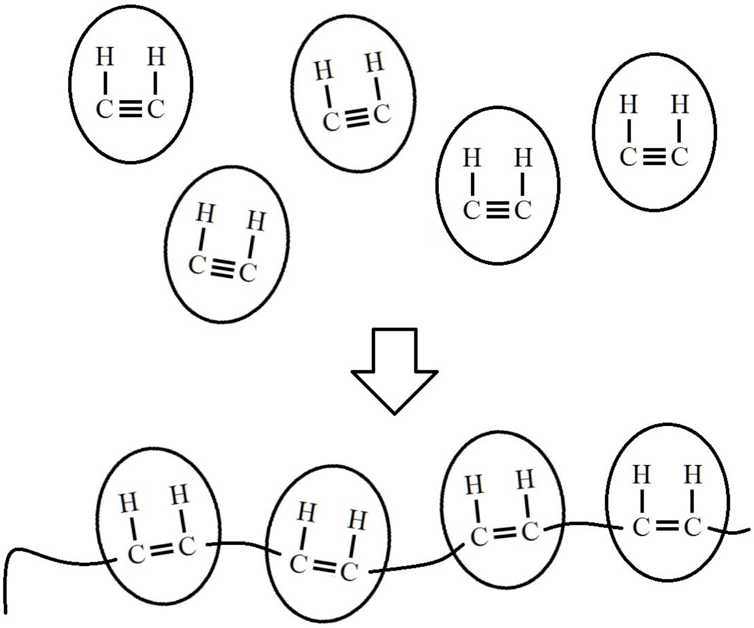


Fig. 11. The idea of connecting the phase transitions with collections of 2-Carbon-2-Hydrogen atoms systems.

The concept is related with the Nobel prize in physics 2016 [21] awarded for theoretical discoveries of topological phase transitions and topological phases of matter.

The polymer molecules, in fact, nanomolecules, in suitable temperature conditions form the so-called molecular nanoengines producing the high energy and probably changing the senary to quinary structure and vice versa. The problem is related to that of Nobel Prize in Chemistry 2016 awarded for the design and the synthesis of molecular machines [18].

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## STRUKTURY BINARNE I TERNARNE W FIZYCE III
## TEORIA GALOIS DLA STRUKTUR BINARNYCH I TERNARNYCH

S t r e s z c z e n i e

Przedstawiona jest teoria typu Galois dla maszyny Turinga wraz z jej odpowiednikiem dla struktur binarnych i ternarnych w fizyce. Ponadto zaprezentowana idea binarno-ternarnej dekompozycji struktur kwinarnych i senarnych w zastosowaniu do badań nad polimerami.

*Słowa kluczowe:* nieprzemienne rozszerzenie Galois, algebry skończenie wymiarowe, pierścienie i algebry łączne, binarne struktury fizyczne, ternarne struktury fizyczne, kwinarne struktury fizyczne, senarne struktury fizyczne