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Dedicated to the memory of
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INEQUALITY FOR THE INNER RADII OF SYMMETRIC NON-OVERLAPPING DOMAINS

Summary

The paper deals with the following problem stated in [1] by V.N. Dubinin and earlier in different form by G.P. Bakhtina [2]. Let $a_0 = 0$, $|a_1| = \ldots = |a_n| = 1$, $a_k \in B_k \subset \overline{\mathbb{C}}$, where $B_0, \ldots, B_n$ are non-overlapping domains, and $B_1, \ldots, B_n$ are symmetric domains about the unit circle. Find the exact upper bound for $r^\gamma(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k)$, where $r(B_k, a_k)$ is the inner radius of B_k with respect to a_k. For $\gamma = 1$ and $n \geq 2$ this problem was solved by L.V. Kovalev [3, 4]. In the present paper it is solved for $\gamma_n = 0,25n^2$ and $n \geq 4$ under the additional assumption that the angles between neighboring line segments $[0, a_k]$ do not exceed $2\pi/\sqrt{2\gamma}$.

Keywords and phrases: inner radius of domain, non-overlapping domains, radial system of points, separating transformation, quadratic differential, Green's function

In geometric function theory of a complex variable problems maximizing the product of inner radii of non-overlapping domains are well known [1–10]. One of the such problems is considered in the article.

Let $\mathbb{N}$, $\mathbb{R}$ be a sets of natural and real numbers, respectively, $\mathbb{C}$ be a complex plane, $\overline{\mathbb{C}} = \mathbb{C} \bigcup \{\infty\}$ be an expanded complex plain or a sphere of Riemann, $\mathbb{R}^+ = (0, \infty)$. Let $r(B, a)$ be the inner radius of the domain $B \subset \overline{\mathbb{C}}$ with respect to the point $a \in B$ (see, f.e. [1–5]). The inner radius of the domain B is associated with the generalized

Green function $g_B(z,a)$ of the domain B by the relations

$$g_B(z,a) = -\ln|z-a| + \ln r(B,a) + o(1), \quad z \to a,$$

$$g_B(z,\infty) = \ln|z| + \ln r(B,a) + o(1), \quad z \to \infty.$$

Let U_1 be a unit circle $|w| \leq 1$.

The system of non-overlapping domains is called a finite set of arbitrary domains $\{B_k\}_{k=0}^{n}$, $n \in \mathbb{N}$, $n \geq 2$ such that $B_k \subset \overline{\mathbb{C}}$, $B_k \cap B_m = \emptyset$, $k \neq m$, $k, m = \overline{0,n}$.

Further we consider the following system of points $A_n := \{a_k \in \mathbb{C}, k = \overline{1,n}\}$, $n \in \mathbb{N}$, $n \geq 2$, satisfying the conditions $|a_k| \in \mathbb{R}^+$, $k = \overline{1,n}$ and $0 = \arg a_1 < \arg a_2 < \dots < \arg a_n < 2\pi$. Denote by

$$P_k = P_k(A_n) := \{w : \arg a_k < \arg w < \arg a_{k+1}\}, \quad a_{n+1} := a_1,$$

$$\alpha_k := \frac{1}{\pi} \arg \frac{a_{k+1}}{a_k}, \quad \alpha_{n+1} := \alpha_1, \quad k = \overline{1,n}, \quad \sum_{k=1}^{n} \alpha_k = 2.$$

Consider the following problem.

Problem. Let $a_0 = 0$, $|a_1| = \ldots = |a_n| = 1$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0,n}$, where $B_0, \ldots, B_n$ are pairwise non-overlapping domains and $B_1, \ldots, B_n$ are symmetric domains with respect to the unit circle. Find the exact upper bound of the product

$$I_n(\gamma) = r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k).$$

For $\gamma = 1$ the problem was formulated as an open problem in the paper [1]. L.V. Kovalev solved the problem for $n \geq 2$ and $\gamma = 1$ [3, 4]. The following theorem substantially complements the results of the papers [2, 3, 4].

Theorem 1. *Let $n \in \mathbb{N}$, $n \geq 2$, $\gamma \in (0, \gamma_n]$, $\gamma_2 = 1,49$, $\gamma_3 = 3,01$, $\gamma_n = 0,25\, n^2$, $n \geq 4$. Then for any different points of a unit circle $|w| = 1$ such that $0 < \alpha_k \leq 2/\sqrt{2\gamma}$, $k = \overline{1,n}$ and for any different system of non-overlapping domains B_k, $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{1,n}$, where the domains B_k, $k = \overline{1,n}$, have symmetry with respect to the unit circle $|w| = 1$, the following inequality holds*

$$r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k) \leq \left(\frac{4}{n}\right)^{n} \frac{\left(\frac{2\gamma}{n^2}\right)^{\frac{\gamma}{n}}}{\left|1 - \frac{2\gamma}{n^2}\right|^{\frac{n}{2} + \frac{\gamma}{n}}} \left|\frac{n - \sqrt{2\gamma}}{n + \sqrt{2\gamma}}\right|^{\sqrt{2\gamma}}. \tag{1}$$

Equality in this inequality is achieved when a_k and B_k, $k = \overline{0,n}$, are, respectively, poles and circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{\gamma w^{2n} + 2(n^2 - \gamma)w^n + \gamma}{w^2(w^n - 1)^2}\, dw^2. \tag{2}$$

Proving the theorem 1. Consider the system of functions

$$\pi_k(w) = \left(e^{-i\arg a_k} w\right)^{\frac{1}{\alpha_k}}, \quad k = \overline{1,n}.$$

The family of functions $\{\pi_k(w)\}_{k=1}^n$ is called admissible for separating transformation of domains B_k, $k = \overline{0,n}$ with respect to angles $\{P_k\}_{k=1}^n$.

Let $\Omega_k^{(1)}$, $k = \overline{1,n}$, denote the domain of the plane $\mathbb{C}_\zeta$, obtained as a result of the union of the connected component of the set $\pi_k(B_k \bigcap \overline{P}_k)$, containing the point $\pi_k(a_k)$ with the own symmetric reflection with respect to the real axis. In turn, by $\Omega_k^{(2)}$, $k = \overline{1,n}$, one denotes the domain of the plain $\mathbb{C}_\zeta$, which are obtained as a result of the union of the connected component of the set $\pi_k(B_{k+1} \bigcap \overline{P}_k)$, containing the point $\pi_k(a_{k+1})$ with the own symmetric reflection with respect to the real axis, $B_{n+1} := B_1$, $\pi_n(a_{n+1}) := \pi_n(a_1)$. Moreover, we denote $\Omega_k^{(0)}$ as the domain of the plane $\mathbb{C}_\zeta$, obtained as a result of the union of the connected component of the set $\pi_k(B_0 \bigcap \overline{P}_k)$ containing the point $\zeta = 0$ with the own symmetric reflection with respect to the real axis. Denote by

$$\pi_k(a_k) := \omega_k^{(1)} = 1, \quad \pi_k(a_{k+1}) := \omega_k^{(2)} = -1, \quad k = \overline{1,n}.$$

From the definition of the function π_k, it follows that

$$|\pi_k(w) - 1| \sim \frac{1}{\alpha_k} \cdot |w - a_k|, \quad w \to a_k, \quad w \in \overline{P_k},$$

$$|\pi_k(w) + 1| \sim \frac{1}{\alpha_k} \cdot |w - a_{k+1}|, \quad w \to a_{k+1}, \quad w \in \overline{P_k},$$

$$|\pi_k(w)| \sim |w|^{\frac{1}{\alpha_k}}, \quad w \to 0, \quad w \in \overline{P_k}.$$

Further, using the result of the papers [1, 2], we obtain the inequalities

$$r\left(B_k, a_k\right) \le \left[\alpha_k r\left(\Omega_k^{(1)}, 1\right) \cdot \alpha_{k-1} r\left(\Omega_k^{(2)}, -1\right)\right]^{\frac{1}{2}}, \quad k = \overline{1,n}, \tag{3}$$

$$r\left(B_0, 0\right) \le \left[\prod_{k=1}^{n} r^{\alpha_k^2}\left(\Omega_k^{(0)}, 0\right)\right]^{\frac{1}{2}}. \tag{4}$$

From inequalities (3) and (4), and using the technique developed in [5, p. 269–274], we obtain

$$I_n(\gamma) \le \prod_{k=1}^{n} \left[r\left(\Omega_k^{(0)}, 0\right)\right]^{\frac{\gamma\alpha_k^2}{2}} \prod_{k=1}^{n} \left[\alpha_{k-1} r\left(\Omega_k^{(2)}, -1\right) \alpha_k r\left(\Omega_k^{(1)}, 1\right)\right]^{\frac{1}{2}} =$$

$$= \left(\prod_{k=1}^{n} \alpha_k\right) \left[\prod_{k=1}^{n} r^{\gamma\alpha_k^2}\left(\Omega_k^{(0)}, 0\right) r\left(\Omega_k^{(1)}, 1\right) r\left(\Omega_k^{(2)}, -1\right)\right]^{\frac{1}{2}}. \tag{5}$$

Further, consider the product of three domains

$$r^{\gamma\alpha_k^2}(G_0, 0) r(G_1, 1) r(G_2, -1).$$

To the domains G_0, G_1, G_2 we again apply separating transformation. Let

$$T_k := \{z : (-1)^{k+1} Im\, z > 0\}, \quad k \in \{1, 2\},$$

$$D_1 = \overline{T_1} \cap U_1, \quad D_2 = \overline{\mathbb{C}} \backslash U_1 \cap \overline{T_1}, \quad D_3 = \overline{T_2} \cap U_1, \quad D_4 = \overline{\mathbb{C}} \backslash U_1 \cap \overline{T_2},$$

$$\beta(z) = \frac{2z}{1+z^2}.$$

From the definition of the function $\beta(z)$, it follows that

$$|\beta(z)| \sim 2|z|, \quad z \to 0, \quad z \in \overline{T_k},$$

$$|\beta(z) - 1| \sim \frac{1}{2}\,|z-1|^2, \quad z \to 1, \quad z \in \overline{T_k},$$

$$|\beta(z) + 1| \sim \frac{1}{2}\,|z+1|^2, \quad z \to -1, \quad z \in \overline{T_k}.$$

The result of separating transformation the domain G_0 with respect to the function $\beta(z)$ and the system of domains $\{\overline{D}_k\}_{k=1}^4$ denote by $G_0^{(k)}$, $k = \overline{1,4}$; besides, the result of separating transformation the domain G_j, $j \in \{1, 2\}$, with respect to the function $2z/(1+z^2)$ and the system of domains $\{\overline{D}_k\}_{k=1}^4$ denote by $G_1^{(k)}, G_2^{(k)}$, $k = \overline{1,4}$. Further, we obtain the inequalities

$$r\left(G_0, 0\right) \le \left[\frac{1}{2} r\left(G_0^{(1)}, 0\right) \cdot \frac{1}{2} r\left(G_0^{(2)}, 0\right)\right]^{\frac{1}{2}},$$

$$r\left(G_1, 1\right) \le \left[2r\left(G_1^{(1)}, 1\right) 2r\left(G_1^{(2)}, 1\right) 2r\left(G_1^{(3)}, 1\right) 2r\left(G_1^{(4)}, 1\right)\right]^{\frac{1}{8}},$$

$$r\left(G_2, -1\right) \le \left[2r\left(G_2^{(1)}, -1\right) 2r\left(G_2^{(2)}, -1\right) 2r\left(G_2^{(3)}, -1\right) 2r\left(G_2^{(4)}, -1\right)\right]^{\frac{1}{8}}.$$

Since the domains G_1, G_2, have symmetry with respect to the unit circle, then

$$r^{\alpha_k^2\gamma}\left(G_0, 0\right) r\left(G_1, 1\right) r\left(G_2, -1\right) \le$$

$$\le 2^{1-\alpha_k^2\gamma}\left[r^{2\alpha_k^2\gamma}\left(G_0^{(1)}, 0\right) r\left(G_1^{(1)}, 1\right) r\left(G_2^{(1)}, -1\right)\right]^{\frac{1}{2}} \times$$

$$\times \left[r^{2\alpha_k^2\gamma}\left(G_0^{(3)}, 0\right) r\left(G_1^{(3)}, 1\right) r\left(G_2^{(3)}, -1\right)\right]^{\frac{1}{2}}.$$

In case $2\alpha_k^2\gamma \le 4$, using paper [1], we obtain

$$r^{2\alpha_k^2\gamma}\left(G_0^{(s)}, 0\right) r\left(G_1^{(s)}, 1\right) r\left(G_2^{(s)}, -1\right) \le$$

$$\le \frac{2^{2\gamma\alpha_k^2+6}(\alpha_k\sqrt{2\gamma})^{2\gamma\alpha_k^2}}{(2-\alpha_k\sqrt{2\gamma})^{\frac{1}{2}(2-\alpha_k\sqrt{2\gamma})^2}(2+\alpha_k\sqrt{2\gamma})^{\frac{1}{2}(2+\alpha_k\sqrt{2\gamma})^2}}, \quad s \in \{1, 3\}.$$

Equality in this inequality is achieved when $G_0^{(s)}$, $G_1^{(s)}$, $G_2^{(s)}$ are circular domains of the quadratic differential

$$Q(z)dz^2 = -\frac{(4-2\alpha_k^2\gamma)z^2 + 2\alpha_k^2\gamma}{z^2(z^2-1)^2}\,dz^2 \tag{6}$$

($0\in G_0^{(s)}$, $1\in G_1^{(s)}$, $-1\in G_2^{(s)}$, $s\in\{1,3\}$). Since $\alpha_k^2\gamma\le 2$, then according to the papers [1, 6], the inequality holds

$$r^{\gamma}(B_0,0)\prod_{k=1}^{n}r(B_k,a_k)\le\left(\frac{1}{\sqrt{2\gamma}}\right)^{n}\prod_{k=1}^{n}\left(\alpha_k\sqrt{2\gamma}\right)2^{\frac{1-\alpha_k^2\gamma}{2}}\times$$
$$\times\left[\frac{2^{2\gamma\alpha_k^2+6}(\alpha_k\sqrt{2\gamma})^{2\gamma\alpha_k^2}}{(2-\alpha_k\sqrt{2\gamma})^{\frac{1}{2}(2-\alpha_k\sqrt{2\gamma})^2}(2+\alpha_k\sqrt{2\gamma})^{\frac{1}{2}(2+\alpha_k\sqrt{2\gamma})^2}}\right]^{\frac{1}{4}}=$$
$$=\left(\frac{1}{\sqrt{2\gamma}}\right)^{n}\prod_{k=1}^{n}\left[\frac{2^{8}\,(\alpha_k\sqrt{2\gamma})^{2\gamma\alpha_k^2+4}}{(2-\alpha_k\sqrt{2\gamma})^{\frac{1}{2}(2-\alpha_k\sqrt{2\gamma})^2}(2+\alpha_k\sqrt{2\gamma})^{\frac{1}{2}(2+\alpha_k\sqrt{2\gamma})^2}}\right]^{\frac{1}{4}}.$$

Consider the function

$$\Psi(x)=2^8\cdot x^{x^2+4}\cdot(2-x)^{-\frac{1}{2}(2-x)^2}\cdot(2+x)^{-\frac{1}{2}(2+x)^2},$$

where $x=\alpha_k\sqrt{2\gamma}$, $x\in(0,2]$.

Consider the extremal problem

$$\prod_{k=1}^{n}\Psi(x_k)\longrightarrow\max,\quad \sum_{k=1}^{n}x_k=2\sqrt{2\gamma},$$
$$x_k=\alpha_k\sqrt{2\gamma},\quad 0<x_k\le 2.$$

Let $F(x)=\ln(\Psi(x))$ and $X^{(0)}=\left\{x_k^{(0)}\right\}_{k=1}^{n}$ is any extremal point above the indicated problem. Repeating the arguments of [6], we obtain the statement: if $0<x_k^{(0)}<x_j^{(0)}<2$, then the following equalities hold $F'(x_k^{(0)})=F'(x_j^{(0)})$, and when some $x_j^{(0)}=2$, then for any $x_k^{(0)}<2$, $F'(x_k^{(0)})\le F'(2)$, where $k,j=\overline{1,n}$, $k\ne j$,

$$F'(x)=2x\ln x+(2-x)\ln(2-x)-(2+x)\ln(2+x)+\frac{4}{x}$$

(see. Fig. 1).

We verify that assertion is correct: if the function $Z(x_1,\ldots,x_n)=\sum\limits_{k=1}^{n}F(x_k)$ reaches a maximum at the point $(x_1^{(0)},\ldots,x_n^{(0)})$ with conditions $0<x_k^{(0)}\le 2$, $k=\overline{1,n}$, $\sum\limits_{k=1}^{n}x_k^{(0)}=2\sqrt{2\gamma}$, then

$$x_1^{(0)}=x_2^{(0)}=\ldots=x_n^{(0)}.$$

For the simplicity, let $x_1^{(0)}\le x_2^{(0)}\le\ldots\le x_n^{(0)}$. The function

$$F''(x)=\ln\left(\frac{x^2}{4-x^2}\right)-\frac{4}{x^2}$$

strictly ascending by $(0,2)$ and exist $x_0\approx 1,768828$ such that

$$sign F''(x)\equiv sign(x-x_0).$$

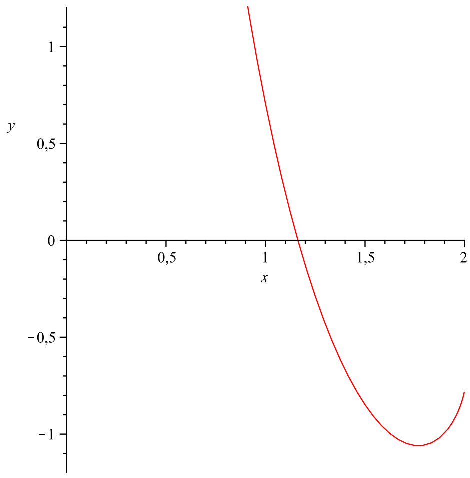

Fig. 1. Graph of the function $y = F'(x)$

Taking into consideration properties of the function $F'(x)$, the condition of theorem and relying on the method developed in [6], we obtain that for $F'(x)$ the inequality $(x_1 - 1,45)\,n + (x_2 - x_1) > 0$ always holds for $n \geq 4$. Hence $nx_1 + (x_2 - x_1) > 1,45\,n$. And, finally, we get

$$(n-1)x_1 + x_2 > 1,45n = 2\sqrt{2\gamma_n}, \quad \gamma_n = 0,25\,n^2, \quad n \geq 4.$$

So, in the case $n \geq 4$ the set of points $\left\{x_k^{(0)}\right\}_{k=1}^{n}$ can not be extreme under the condition $x_n^{(0)} \in (x_0,\, 2]$. Thus, for an extreme set $\left\{x_k^{(0)}\right\}_{k=1}^{n}$ is possible only the case when $x_k^{(0)} \in (0,\, x_0]$, $k = \overline{1,n}$, and $x_1^{(0)} = x_2^{(0)} = \ldots = x_n^{(0)}$. For any $\gamma < \gamma_n$, $n \geq 4$, all previous arguments remain.

Further, let $F'(x) = t$, $y_0 \leq t \leq -0,78$, $y_0 \approx -1,059$. Consider the following values t: $t_1 = -0,78$, $t_2 = -0,80$, $t_3 = -0,85$, $t_4 = -0,90$, $\cdots$, $t_{11} = -1,05$, $t_{12} = -1,059$. One finds the solution of equation $F'(x) = t_k$, $k = \overline{1,12}$. For any $t_k \in [y_0, -0,78)$ the equation has two solutions: $x_1(t) \in (0, x_0]$, $x_2(t) \in (x_0, 2]$, $x_0 \approx 1,768828$. Direct calculations are presented in the table below.

Taking into consideration properties of the function $F'(x)$ and the condition of theorem, we obtain the following inequality

$$\sum_{k=1}^{n} x_k(t) > (n-1)x_1(t_k) + x_2(t_{k+1}) \geq$$

$$\geq \min_{1\leq k\leq 11} \left((n-1)x_1(t_k) + x_2(t_{k+1})\right) = 2\sqrt{2\gamma_n},$$

where $t_k \leq t \leq t_{k+1}$, $k = \overline{1,11}$. So, we have that for the extremal set $X^{(0)}$ the only case is possible where $\left\{x_k^{(0)}\right\}_{k=1}^{n} \in (0,\, x_0]$, $x_0 \approx 1,7688283$, and therefore $x_1^{(0)} = x_2^{(0)} = \cdots = x_n^{(0)}$.

k	t_k	$x_1(t_k)$	$x_2(t_k)$	$x_1(t_k)+x_2(t_{k+1})$	$2x_1(t_k)+x_2(t_{k+1})$
1	-0,78	1,458417	1,998914		
2	-0,80	1,470034	1,994779	3,453196	4,911613
3	-0,85	1,501193	1,980165	3,450199	4,920233
4	-0,90	1,536275	1,959964	3,461157	4,962350
5	-0,95	1,577242	1,932788	3,469063	5,005338
6	-1,00	1,628755	1,894239	3,471481	5,048723
7	-1,01	1,641325	1,884177	3,512932	5,141687
8	-1,02	1,655169	1,872815	3,514140	5,155465
9	-1,03	1,670801	1,859641	3,514810	5,169979
10	-1,04	1,689217	1,843656	3,514457	5,185258
11	-1,05	1,712998	1,822285	3,511502	5,200719
12	-1,059	1,768589	1,769066	3,482064	5,195062

Finally, we have the relation

$$r^{\gamma}(B_0,0)\prod_{k=1}^{n} r(B_k,a_k) \leq \left(\frac{1}{\sqrt{2\gamma}}\right)^{n}\left[\Psi\left(\frac{2}{n}\sqrt{2\gamma}\right)\right]^{\frac{n}{4}}.$$

Using the specific expression for $\Psi(x)$ and simple transformations, we obtain the inequality (1). In this way, the main inequality of theorem 1 is proved. Realizing in (6) the change of variable by the formula $z = 2w^{\frac{n}{2}}/(1+w^n)$, we obtain the quadratic differential (2). The equality sign is verified directly. The theorem 1 is proved.

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NIERÓWNOŚĆ NA WEWNĘTRZNE PROMIENIE SYMETRYCZNYCH NIE ZACHODZĄCYCH NA SIEBIE OBSZARÓW

S t r e s z c z e n i e

Praca dotyczy zagadnienia postawionego przez V. N. Dubinina [1], a przedtem w innej postaci przez G. P. Bakhtina [2]. Niech $a_0 = 0$, $|a_1| = \ldots = |a_n| = 1$, $a_k \in B_k \subset \overline{\mathbb{C}}$, gdzie $B_0, \ldots, B_n$ są nie zachodzącymi na siebie obszarami, przy czym obszary $B_1, \ldots, B_n$ są symetryczne względem okręgu jednostkowego. Problem polega na znalezieniu dokładnego kresu górnego dla $r^\gamma(B_0, 0) \prod\limits_{k=1}^{n} r(B_k, a_k)$, gdzie $r(B_k, a_k)$ jest promieniem wewnętrznym obszaru B_k względem punktu a_k. Dla $\gamma = 1$ i $n \geq 2$ problem został rozwiązany przez L.V. Kovaleva [3, 4]. W obecnej pracy problem został rozwiązany dla $\gamma_n = 0,25n^2$ i $n \geq 4$ przy dodatkowym założeniu, że kąty między sąsiednimi odcinkami $[0, a_k]$ nie przekraczają $2\pi/\sqrt{2\gamma}$.

Słowa kluczowe: promień wewnętrzny obszaru, obszary nie zachodzące na siebie, promienisty układ punktów, transformacja rozdzielająca, różniczka kwadratowa, funkcja Greena