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GLOBAL FINITE MEAN OSCILLATION AND THE BELTRAMI EQUATION

Abstract

In this paper, the estimate for growth of homeomorphic solutions of the Beltrami equation at infinity is obtained, provided that the dilatation quotient has a global finite mean oscillation.

Keywords and phrases: Beltrami equations, ring Q–homeomorphisms, modulus, capacity.
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1 Introduction

Let D be a domain in the complex plane $\mathbb{C}$, i.e., a connected and open subset of $\mathbb{C}$, and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a.e. (almost everywhere) in D. The *Beltrami equation* is the equation of the form

$$f_{\overline{z}} = \mu(z) f_z \tag{1}$$

where $f_{\overline{z}} = \overline{\partial} f = (f_x + i f_y)/2$, $f_z = \partial f = (f_x - i f_y)/2$, $z = x + iy$, and f_x and f_y are partial derivatives of f in x and y, correspondingly. The function μ is called the *complex coefficient* and

$$K_\mu(z) = \frac{1 + |\mu(z)|}{1 - |\mu(z)|} \tag{2}$$

the *dilatation quotient* for the equation (1). The Beltrami equation (1) is said to be *degenerate* if $\operatorname{ess\,sup} K_\mu(z) = \infty$. The existence theorem for homeomorphic $W^{1,1}_{\rm loc}$ solutions was established to many degenerate Beltrami equations, see, e.g., related references in the recent monographs [3], [10], [7]; cf. also [6], [14] – [18].

Recall that the *(conformal) modulus* of a family Γ of curves γ in $\mathbb{C}$ is the quantity

$$M(\Gamma) = \inf_{\rho \in \operatorname{adm} \Gamma} \int\limits_{\mathbb{C}} \rho^2(z) \ \ dx\, dy \tag{3}$$

where adm Γ is the class of all Borel functions $\rho : \mathbb{C} \to [0, \infty]$ such that

$$\int\limits_{\gamma} \rho \, ds \geqslant 1 \qquad \forall\, \gamma \in \Gamma, \tag{4}$$

where s is the arc length parametrization of γ.

Throughout this paper,

$$B(z_0, r) = \{z \in \mathbb{C} : |z - z_0| < r\} ,$$

$$S(z_0, r) = \{z \in \mathbb{C} : |z - z_0| = r\} ,$$

and

$$\mathbb{A}(z_0, r_1, r_2) = \{z \in \mathbb{C} : r_1 < |z - z_0| < r_2\} .$$

Let $E, F \subset \overline{\mathbb{C}}$ be arbitrary sets. Denote by $\Delta(E, F, D)$ a family of all curves $\gamma : [a, b] \to \overline{\mathbb{C}}$ joining E and F in D, i.e., $\gamma(a) \in E, \gamma(b) \in F$ and $\gamma(t) \in D$ as $t \in (a, b)$.

Here a *condenser* is a pair $\mathcal{E} = (A, C)$ where $A \subset \mathbb{C}$ is open and C is a non-empty compact set contained in A. $\mathcal{E}$ is a *ringlike condenser* if $B = A \setminus C$ is a ring, i.e., if B is a domain whose complement $\overline{\mathbb{C}} \setminus B$ has exactly two components where $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ is the one-point compactification of $\mathbb{C}$. $\mathcal{E}$ is a *bounded condenser* if A is bounded. A condenser $\mathcal{E} = (A, C)$ is said to be in a domain G if $A \subset G$.

The following lemma is immediate:

Lemma 1.1. *If $f : G \to \mathbb{C}$ is a homeomorphism and $\mathcal{E} = (A, C)$ is a condenser in G, then (fA, fC) is a condenser in fG.*

In the above situation we denote $f\mathcal{E} = (fA, fC)$.

Let $\mathcal{E} = (A, C)$ be a condenser. We set

$$\operatorname{cap} \mathcal{E} = \operatorname{cap} (A, C) = \inf_{u \in W_0(\mathcal{E})} \int\limits_{A} |\nabla u|^2 \, dxdy$$

and call it the *capacity* of the condenser $\mathcal{E}$. The set $W_0(\mathcal{E}) = W_0(A, C)$ is the family of nonnegative functions $u : A \to \mathbb{R}$ such that $u \in C_0(A)$, $u(z) \geqslant 1$ for $z \in C$, and u is absolutely continuous on lines (ACL). In the above formula,

$$|\nabla u| = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2} .$$

We mention some properties of the capacity of a condenser. It was proven in ([20], Theorem 1) that

$$\operatorname{cap} \mathcal{E} = M(\Delta(\partial A, \partial C; A \setminus C)) , \tag{5}$$

where $\Delta(\partial A, \partial C; A \setminus C)$ denotes the set of all continuous curves joining the boundaries ∂A and ∂C in $A \setminus C$.

Moreover, the following estimate is known:

$$\operatorname{cap}\mathcal{E} \geqslant \frac{4\pi}{\log\frac{m(A)}{m(C)}} \tag{6}$$

(see, e.g., (8.8) in [11]).

The following notion is motivated by the ring definition of Gehring for quasiconformal mappings, see, e.g., [5], introduced first in the plane, see [17], and extended later on to the space case in [13], see also Chapters 7 and 11 in [10], cf. [1], [2], [4], [12].

Given a domain D in $\mathbb{C}$, a (Lebesgue) measurable function $Q : D \to [0,\infty]$, $z_0 \in D$, a homeomorphism $f : D \to \overline{\mathbb{C}}$ is said to be a *ring Q–homeomorphism at the point z_0* if

$$M\left(f\left(\Delta\left(S_1,\, S_2,\, \mathbb{A}(z_0,r_1,r_2)\right)\right)\right) \leqslant \int\limits_{\mathbb{A}(z_0,r_1,r_2)} Q(z)\cdot\eta^2(|z-z_0|)\, dx\, dy \tag{7}$$

for every ring $\mathbb{A}(z_0,r_1,r_2)$ and the circles $S_i = S(z_0,r_i)$, $i=1,2$, where $0<r_1<r_2<r_0 :\ = \operatorname{dist}(z_0,\partial D)$, and every measurable function $\eta : (r_1,r_2)\to[0,\infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)\; dr \;=\; 1.$$

The homeomorphism f is called a *ring Q–homeomorphism in the domain D* if f is a ring Q–homeomorphism at every point $z_0 \in D$.

The following statement was first proved in [9], Theorem 3.1, cf. also Corollory 3.1 in [19].

Proposition 1.2. *Let f be a homeomorphic $W^{1,1}_{\rm loc}$ solution of the Beltrami equation (1). Then f is a ring Q-homeomorphism at each point $z_0 \in D$ with $Q(z) = K_\mu(z)$.*

2 GFMO functions

Similarly to [8] (cf. also [14], [16]), we say that a function $\varphi\colon \mathbb{C}\to\mathbb{R}$ has *global finite mean oscillation at a point $z_0\in\mathbb{C}$*, abbr. $\varphi \in GFMO(z_0)$, if

$$\limsup_{R\to\infty} \frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z)-\overline{\varphi}_R|\, dxdy < \infty, \tag{8}$$

where

$$\overline{\varphi}_R = \frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} \varphi(z)\, dxdy$$

is the mean value of the function $\varphi(z)$ over $B(z_0,R)$, $R>0$. Here $B(z_0,R)=\{z\in\mathbb{C}\colon |z-z_0|<R\}$, and condition (8) includes the assumption that φ is integrable in $B(z_0,R)$ for $R>0$.

Proposition 2.1. *If, for some collection of numbers $\varphi_R \in \mathbb{R}$, $R \in [r_0, +\infty)$, $r_0 > 0$,*

$$\limsup_{R\to\infty} \frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z) - \varphi_R|\, dxdy < \infty,$$

then φ has global finite mean oscillation at z_0.

Proof. Indeed, by the triangle inequality,

$$\frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z) - \overline{\varphi}_R|\, dxdy \leqslant$$

$$\leqslant \frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z) - \varphi_R|\, dxdy + |\varphi_R - \overline{\varphi}_R| \leqslant$$

$$\leqslant \frac{2}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z) - \varphi_R|\, dxdy.$$

□

Corollary 2.2. *If, for a point $z_0 \in \mathbb{C}$,*

$$\limsup_{R\to\infty} \frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z) - \varphi(z_0)|\, dxdy < \infty,$$

then φ has global finite mean oscillation at z_0.

Corollary 2.3. *If, for a point $z_0 \in \mathbb{C}$,*

$$\limsup_{R\to\infty} \frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z)|\, dxdy < \infty,$$

then φ has global finite mean oscillation at z_0.

Lemma 2.4. *Let $z_0 \in \mathbb{C}$. If a nonnegative function $\varphi\colon \mathbb{C} \to \mathbb{R}$ has global finite mean oscillation at z_0 and φ is integrable in $B(z_0, e)$, then, for $R > e^e$,*

$$\int\limits_{\mathbb{A}(z_0,e,R)} \frac{\varphi(z)\, dxdy}{\left(|z - z_0| \log |z - z_0|\right)^2} \leqslant C \cdot \log\log R,$$

where

$$C = \frac{\pi}{6}((24 + \pi^2)e^2\delta_\infty + 2\pi^2\varphi_0),$$

φ_0 is the mean value of φ over the disk $B(z_0, e)$ and

$$\delta_\infty = \delta_\infty(\varphi) = \sup_{R\in(e,+\infty)} \frac{1}{m\left(B(z_0,R)\right)} \int\limits_{B(z_0,R)} |\varphi(z) - \overline{\varphi}_R|\, dxdy$$

is the maximal dispersion of φ.

Proof. Let $R > e^e$, $r_k = e^k$, $\mathbb{A}_k = \{z \in \mathbb{C}\colon r_k \leqslant |z - z_0| < r_{k+1}\}$. Clearly,

$$\delta_\infty = \sup_{R \in (e,\, +\infty)} \frac{1}{m\left(B(z_0, R)\right)} \int\limits_{B(z_0,R)} |\varphi(z) - \overline{\varphi}_R| \, dxdy < \infty \,,$$

$B_k = B(z_0, r_k)$ and let φ_k be the mean value of $\varphi(z)$ over B_k, $k = 1, 2, \ldots$. Take a natural number N such that $R \in [r_N, r_{N+1})$. Then

$$\mathbb{A}(z_0, e, R) \subset \Delta(R) = \bigcup_{k=1}^{N} \mathbb{A}_k$$

and

$$I(R) = \int\limits_{\Delta(R)} \varphi(z)\alpha(|z - z_0|) \, dxdy \leqslant |S_1(R)| + S_2(R) \,,$$

$$\alpha(t) = \frac{1}{(t \log t)^2} \,,$$

$$S_1(R) = \sum_{k=1}^{N} \int\limits_{\mathbb{A}_k} (\varphi(z) - \varphi_{k+1})\alpha(|z - z_0|) \, dxdy \,,$$

and

$$S_2(R) = \sum_{k=1}^{N} \varphi_{k+1} \int\limits_{\mathbb{A}_k} \alpha(|z - z_0|) \, dxdy \,.$$

Since $\mathbb{A}_k \subset B_{k+1}$, $\frac{1}{|z-z_0|^2} \leqslant \frac{\pi e^2}{m(B_{k+1})}$ for $z \in \mathbb{A}_k$ and $\log |z - z_0| > k$ in $\mathbb{A}_k$, then

$$|S_1(R)| \leqslant \pi e^2 \sum_{k=1}^{N} \frac{1}{k^2} \cdot \frac{1}{m(B_{k+1})} \int\limits_{B_{k+1}} |\varphi(z) - \varphi_{k+1}| \, dxdy \leqslant$$

$$\leqslant \pi e^2 \delta_\infty \sum_{k=1}^{N} \frac{1}{k^2} \leqslant \pi e^2 \delta_\infty \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^3 e^2 \delta_\infty}{6} \,.$$

Now,

$$\int\limits_{\mathbb{A}_k} \alpha(|z - z_0|) \, dxdy \leqslant \frac{1}{k^2} \int\limits_{\mathbb{A}_k} \frac{dxdy}{|z - z_0|^2} = \frac{2\pi}{k^2} \,.$$

Moreover,

$$|\varphi_{k-1} - \varphi_k| = \left| \frac{1}{m(B_{k-1})} \int\limits_{B_{k-1}} \varphi(z) \, dxdy - \frac{1}{m(B_{k-1})} \int\limits_{B_{k-1}} \varphi_k \, dxdy \right| \leqslant$$

$$\leqslant \frac{1}{m(B_{k-1})} \int\limits_{B_{k-1}} |\varphi(z) - \varphi_k| \, dxdy \leqslant \frac{e^2}{m(B_k)} \int\limits_{B_k} |\varphi(z) - \varphi_k| \, dxdy \leqslant e^2 \delta_\infty \,,$$

and by the triangle inequality, for $k \geqslant 1$

$$\varphi_{k+1} = |\varphi_{k+1}| = \left| \varphi_1 + \sum_{l=2}^{k+1} (\varphi_l - \varphi_{l-1}) \right| \leqslant$$

$$\leqslant |\varphi_1| + \sum_{l=2}^{k+1} |\varphi_l - \varphi_{l-1}| \leqslant |\varphi_1| + e^2 \delta_\infty \, k.$$

Hence,

$$S_2(R) = |S_2(R)| \leqslant 2\pi \sum_{k=1}^{N} \frac{\varphi_{k+1}}{k^2} \leqslant 2\pi \sum_{k=1}^{N} \frac{\varphi_1 + e^2 \delta_\infty \, k}{k^2} \leqslant$$

$$\leqslant 2\pi \varphi_1 \sum_{k=1}^{\infty} \frac{1}{k^2} + 2\pi e^2 \delta_\infty \sum_{k=1}^{N} \frac{1}{k} =$$

$$= \frac{\pi^3 \varphi_1}{3} + 2\pi e^2 \delta_\infty \sum_{k=1}^{N} \frac{1}{k} \, .$$

But

$$\sum_{k=2}^{N} \frac{1}{k} < \int\limits_1^N \frac{dt}{t} = \log N$$

and, for $R > r_N$,

$$N = \log r_N < \log R \, .$$

Consequently,

$$\sum_{k=1}^{N} \frac{1}{k} < 1 + \log \log R$$

and thus, for $R \in (e^e, +\infty)$

$$I(R) \leqslant \frac{\pi^3 e^2 \delta_\infty}{6} + \frac{\pi^3 \varphi_1}{3} + 2\pi e^2 \delta_\infty (1 + \log \log R) =$$

$$= \left(\frac{\pi^3 e^2 \delta_\infty + 12\pi e^2 \delta_\infty + 2\pi^3 \varphi_1}{6 \log \log R} + 2\pi e^2 \delta_\infty \right) \log \log R \leqslant$$

$$\leqslant \frac{\pi}{6} ((24 + \pi^2) e^2 \delta_\infty + 2\pi^2 \varphi_1) \log \log R \, .$$

Finally,

$$\int\limits_{\mathbb{A}(z_0, e, R)} \frac{\varphi(z) \, dxdy}{(|z - z_0| \log |z - z_0|)^2} \leqslant I(R) \leqslant \frac{\pi}{6} ((24 + \pi^2) e^2 \delta_\infty + 2\pi^2 \varphi_1) \log \log R \, .$$

$\square$

3 The behavior at infinity of homeomorphic solutions of the Beltrami equations

Set

$$l_f(z_0,e)=\min_{|z-z_0|=e}|f(z)-f(z_0)|\,,$$

$$\delta_\infty=\delta_\infty\left(K_\mu,z_0\right)=\sup_{R\in(e,+\infty)}\frac{1}{m\left(B(z_0,R)\right)}\int\limits_{B(z_0,R)}|K_\mu(z)-K_{\mu,z_0}(R)|\,dxdy\,,$$

$$K_{\mu,z_0}(R)=\frac{1}{m\left(B(z_0,R)\right)}\int\limits_{B(z_0,R)}K_\mu(z)\,dxdy\,,\quad k_0=K_{\mu,z_0}(e)\,.$$

Theorem 3.1. *Let $\mu\colon\mathbb{C}\to\mathbb{C}$ be a measurable function with $|\mu(z)|<1$ a.e. and $f\colon\mathbb{C}\to\mathbb{C}$ be a homeomorphic $W^{1,1}_{\rm loc}$ solution of the Beltrami equation (1). If $K_\mu\in GFMO(z_0)$, $z_0\in\mathbb{C}$, then*

$$\liminf_{R\to\infty}\frac{\max\limits_{|z-z_0|=R}|f(z)-f(z_0)|}{(\log R)^{\frac{2\pi}{C}}}\geqslant l_f(z_0,e)\,, \tag{9}$$

where $C=\frac{\pi}{6}((24+\pi^2)e^2\delta_\infty+2\pi^2k_0)$.

Proof. Consider the ring $\mathbb{A}(R)=\mathbb{A}(z_0,e,R)$, with $R>e^e$. Set $\mathcal{E}=(B(z_0,R),\,\overline{B(z_0,e)})$. Then, by Lemma 1.1, $f\mathcal{E}=(fB(z_0,R),\,f\overline{B(z_0,e)})$ is a condenser in $\mathbb{C}$, according (5),

$${\rm cap}\,(fB(z_0,R),\,f\overline{B(z_0,e)})=M(\Delta(\partial fB(z_0,e),\partial fB(z_0,R);f\mathbb{A}(R)))$$

and, in view of the homeomorphism of f,

$$\Delta(\partial fB(z_0,e),\partial fB(z_0,R);f\mathbb{A}(R))=f\Delta(\partial B(z_0,e),\partial B(z_0,R);\mathbb{A}(R))\,.$$

By Proposition 1.2, f is a ring Q-homeomorphism with $Q=K_\mu(z)$, and so

$${\rm cap}\,(fB(z_0,R),\,f\overline{B(z_0,e)})\leqslant\int\limits_{\mathbb{A}(R)}K_\mu(z)\eta^2(|z-z_0|)\,dxdy \tag{10}$$

for every measurable function $\eta\colon(e,R)\to[0,+\infty]$ such that

$$\int\limits_e^R\eta(t)\,dt=1.$$

Choosing in (10), $\eta(t)=\frac{1}{t\log t\cdot\log\log R}$, we obtain

$${\rm cap}\,(fB(z_0,R),\,f\overline{B(z_0,e)})\leqslant\frac{1}{(\log\log R)^2}\cdot\int\limits_{\mathbb{A}(R)}\frac{K_\mu(z)\,dxdy}{(|z-z_0|\log|z-z_0|)^2}\,.$$

Since $K_\mu \in GFMO(z_0)$, then by Lemma 2.4,

$$\mathrm{cap}\,(fB(z_0,R),\, f\overline{B(z_0,e)}) \leqslant \frac{C}{\log\log R}\,, \tag{11}$$

where $C=\frac{\pi}{6}((24+\pi^2)e^2\delta_\infty+2\pi^2k_0)$. On the other hand, by (6), we have

$$\mathrm{cap}\,(fB(z_0,R),\, f\overline{B(z_0,e)}) \geqslant \frac{4\pi}{\log\frac{m(fB(z_0,R))}{m(f\overline{B(z_0,e)})}}\,. \tag{12}$$

Combining (11) and (12), we obtain

$$\frac{4\pi}{\log\frac{m(fB(z_0,R))}{m(f\overline{B(z_0,e)})}} \leqslant \frac{C}{\log\log R}\,.$$

This gives

$$m(f\overline{B(z_0,e)}) \leqslant \frac{m(fB(z_0,R))}{(\log R)^{\frac{4\pi}{C}}}\,.$$

Using the inequalities

$$\pi\left(\min_{|z-z_0|=e}|f(z)-f(z_0)|\right)^2 \leqslant m(f\overline{B(z_0,e)}) \leqslant$$

$$\leqslant m(fB(z_0,R)) \leqslant \pi\left(\max_{|z-z_0|=R}|f(z)-f(z_0)|\right)^2\,,$$

we obtain

$$\min_{|z-z_0|=e}|f(z)-f(z_0)| \leqslant \frac{\max\limits_{|z-z_0|=R}|f(z)-f(z_0)|}{(\log R)^{\frac{2\pi}{C}}}\,. \tag{13}$$

Recall that

$$l_f(z_0,e)=\min_{|z-z_0|=e}|f(z)-f(z_0)|\,.$$

Passing to the lower limit as $R\to\infty$ in (13), we obtain the relation (9). $\square$

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Globalna średnia skończona oscylacja i równania Beltramiego

S t r e s z c z e n i e W niniejszej pracy oszacowano wzrost homeomorficznych rozwiązań równania Beltramiego w nieskończoności przy założeniu, że iloraz dylatacji ma globalną skończoną średnią oscylację.

Słowa kluczowe: Równania Beltramiego, Q–homeomorfizmy pierścieniowe, moduł, pojemność.